• Employment Test Prescriptives •

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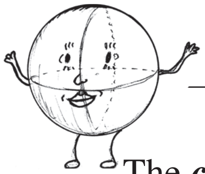
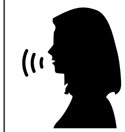


• Probability •



The ***chance*** or ***likelihood*** that something will happen is called **PROBABILITY**.

If something will *definitely* happen, we say its **PROBABILITY** is **CERTAIN**. If something can *never* happen, we say its **PROBABILITY** is **IMPOSSIBLE**.

PROBABILITY can be written as a fraction ($\frac{1}{4}$), a decimal (0.25), or a percent (25%) where the **PROBABILITY** that something is **IMPOSSIBLE** is 0 (or 0%) and the **PROBABILITY** that something is **CERTAIN** is 1 (or 100%). All other **PROBABILITIES** fall in between 0 and 1.

Let's look at the **PROBABILITY** scale below, which shows **PROBABILITIES** with corresponding values.

IMPOSSIBLE		**EQUALLY LIKELY**		**CERTAIN**
0%	⟵ **UNLIKELY** ⟶	50%	⟵ **LIKELY** ⟶	100%
0		($\frac{1}{2}$)		1

CERTAIN: 100%. Will always happen.

LIKELY: Between 50% and 100%. Will probably happen.

EQUALLY LIKELY: 50%. Will happen as often as not.

UNLIKELY: Between 0% and 50%. Will probably not happen.

IMPOSSIBLE: 0%. Will never happen.

Try these:

Based on the probabilities given, classify the likelihood of the event as either CERTAIN, LIKELY, EQUALLY LIKELY, UNLIKELY, or IMPOSSIBLE.

1) The **PROBABILITY** that the sun will rise is 100%. ________________

2) The **PROBABILITY** that it will rain is $\frac{2}{5}$. ________________

3) The **PROBABILITY** that turtles will fly is 0. ________________

4) The **PROBABILITY** that you will see a bird today is 0.84. ________________

• Probability •

Based on the probabilities given, classify the likelihood of the event as either CERTAIN, LIKELY, EQUALLY LIKELY, UNLIKELY, or IMPOSSIBLE.

1) The PROBABILITY that you will win a raffle is 0.08. ____________________

2) The PROBABILITY that a coin will land heads is $\frac{1}{2}$. ____________________

3) The PROBABILITY that it will snow is 71%. ____________________

4) The PROBABILITY that the sun will set is 100%. ____________________

5) The PROBABILITY that you will grow 18 feet is 0%. ____________________

6) The PROBABILITY that you will find a coin is 0.1. ____________________

7) The PROBABILITY that a die will roll a 3 is $\frac{1}{6}$. ____________________

8) The PROBABILITY that you will see a leprechaun is 0. ____________________

9) The PROBABILITY that you will be older tomorrow is 1. ____________________

10) The PROBABILITY that it will not rain is 0.15. ____________________

11) The PROBABILITY that you will lose a pencil is 87%. ____________________

12) The PROBABILITY that you will travel to Spain is 0.5. ____________________

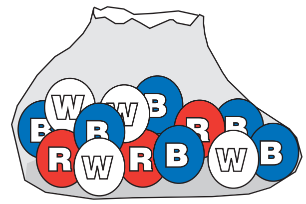
Direct Teaching

Probability equals

$$\frac{\text{The number of ways an event can happen (favorable outcomes)}}{\text{The total number of possible outcomes (possible outcomes)}}$$

Find the **PROBABILITY** of something happening by making a fraction of the number of favorable outcomes *out of* all possible outcomes.

Example 1: A bag contains 3 red marbles, 4 white marbles, and 6 blue marbles. If 1 marble is pulled from the bag at random, what is the probability that the marble is blue?



The number of possible outcomes (total number of marbles) is found by adding up the number of red marbles, white marbles, and blue marbles.

3	+	4	+	6	=	13
RED MARBLES		WHITE MARBLES		BLUE MARBLES		TOTAL MARBLES

Since we are looking for the **PROBABILITY** that the marble is blue, the number of favorable outcomes is found by looking at the number of blue marbles (circled above). We have 6 favorable outcomes.

So, the **PROBABILITY** of pulling a blue marble is the number of favorable outcomes (6) over the number of possible outcomes (13), or $\frac{6}{13}$.

Try this:

1) A bag contains 10 red marbles, 8 white marbles, and 6 blue marbles. If 1 marble is randomly pulled from the bag, what is the **PROBABILITY** the marble is white?

______	+	______	+	______	=	______
RED MARBLES		WHITE MARBLES		BLUE MARBLES		TOTAL MARBLES

So, the **PROBABILITY** of pulling a white marble is _________.

• Probability •

PROBABILITY equals

$$\frac{\text{The number of ways an event can happen (favorable outcomes)}}{\text{The total number of possible outcomes (possible outcomes)}}$$

---

1) A jar contains 16 chocolates, 12 mints, and 8 gumballs. If 1 candy is randomly pulled from the jar, what is the **PROBABILITY** the candy is a mint?

__________ + __________ + __________ = __________

CHOCOLATES MINTS GUMBALLS TOTAL CANDIES

So, the **PROBABILITY** of pulling a mint is __________.

---

2) A closet contains 7 footballs, 15 tennis balls, and 6 basketballs. If 1 ball is randomly pulled from the closet, what is the **PROBABILITY** the ball is a football?

__________ + __________ + __________ = __________

FOOTBALLS TENNIS BALLS BASKETBALLS TOTAL BALLS

So, the **PROBABILITY** of pulling a football is __________.

---

3) A box contains 16 plums, 14 peaches, and 20 apples. If 1 fruit is randomly pulled from the box, what is the **PROBABILITY** the fruit is an apple?

__________ + __________ + __________ = __________

PLUMS PEACHES APPLES TOTAL FRUIT

So, the **PROBABILITY** of pulling an apple is __________.

If we are looking for the **PROBABILITY** that multiple favorable outcomes will occur, we can add together the number of ways that each such outcome can happen.

EXAMPLE:

A toy box contains 5 toy cars, 10 action figures, and 4 dolls. If 1 toy is randomly pulled from the toy box, what is the **PROBABILITY** the toy is an action figure or a doll?

5	+	10	+	4	=	19
TOY CARS		ACTION FIGURES		DOLLS		TOTAL TOYS

Since there are 10 action figures and 4 dolls, there are 14 (10 + 4) favorable outcomes.

So, the **PROBABILITY** of pulling an action figure or a doll is $\frac{14}{19}$.

Try these:

1) A drawer contains 6 blue socks, 5 white socks, and 11 black socks. If 1 sock is randomly pulled from the drawer, what is the **PROBABILITY** the sock is blue or white?

____	+	____	+	____	=	____
BLUE SOCKS		WHITE SOCKS		BLACK SOCKS		TOTAL SOCKS

So, the **PROBABILITY** of pulling a blue or white sock is _________.

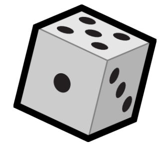
2) A tray contains 14 forks, 9 spoons, and 7 knives. If 1 utensil is randomly pulled from the tray, what is the **PROBABILITY** the utensil is a fork or a knife?

____	+	____	+	____	=	____
FORKS		SPOONS		KNIVES		TOTAL UTENSILS

So, the **PROBABILITY** of pulling a fork or a knife is _________.

• Probability •

A regular die has 6 sides. If 1 die is thrown:



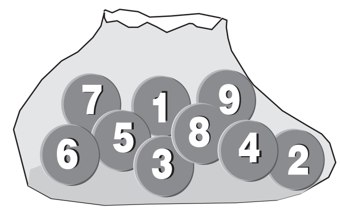
1) What is the PROBABILITY of rolling a 5? ________

2) What is the PROBABILITY of rolling an even number (2, 4, or 6)? ________

3) What is the PROBABILITY of rolling a number less than 3 (1 or 2)? ________

4) What is the PROBABILITY of rolling a 7? ________

A bag contains 9 marbles, numbered 1 through 9. If 1 marble is randomly selected from the bag:



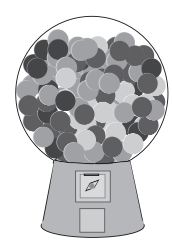
5) What is the PROBABILITY of selecting a 7? ________

6) What is the PROBABILITY of selecting an odd number? ________

7) What is the PROBABILITY of selecting a number greater than 3? ________

8) What is the PROBABILITY of selecting a number less than 10? ________

Mrs. Collins filled an empty gumball machine in her store. She put in 10 blue gumballs, 15 red gumballs, 30 green gumballs, and 20 yellow gumballs.



9) What is the total number of gumballs? ________

If a gumball is randomly selected from the machine:

10) What is the PROBABILITY of getting a yellow gumball? ________

11) What is the PROBABILITY of getting a blue or red gumball? ________

12) What is the PROBABILITY of ***not*** getting a blue gumball? ________

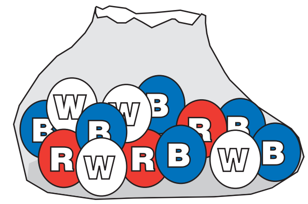
13) What is the PROBABILITY of ***not*** getting a red or green gumball? ________

• Probability Without Replacement •

Direct Teaching

Let's look back at a previous example.

Example 1: A bag contains 3 red marbles, 4 white marbles, and 6 blue marbles. If 1 marble is pulled from the bag at random, what is the **probability** that the marble is blue?



3	+	4	+	6	=	13
RED MARBLES		WHITE MARBLES		BLUE MARBLES		TOTAL MARBLES

Since we are looking for the **probability** that the marble is blue, we have 6 favorable outcomes.

So, the **probability** of pulling a blue marble is the number of favorable outcomes (6) over the number of possible outcomes (13), or $\frac{6}{13}$.

If a blue marble was pulled from the bag and not replaced, what is the **probability** of drawing another blue marble?

				6 – 1		13 – 1
3	+	4	+	5	=	12
RED MARBLES		WHITE MARBLES		BLUE MARBLES		TOTAL MARBLES

Since we did not replace the blue marble, we now only have 5 favorable outcomes and 12 possible outcomes.

So, the **probability** of pulling another blue marble is the number of favorable outcomes (5) over the number of possible outcomes (12), or $\frac{5}{12}$.

Try this:

1) A bag contains 10 red marbles, 8 white marbles, and 6 blue marbles. If a white marble is pulled from the bag and is not put back, what is the **probability** of randomly pulling another white marble?

		8 – 1				
______	+	______	+	______	=	______
RED MARBLES		WHITE MARBLES		BLUE MARBLES		TOTAL MARBLES

So, the **probability** of pulling another white marble is ________.

• Probability •

A closet contains 3 black shirts, 9 white shirts, and 11 green shirts.

1) If a shirt is randomly selected from the closest, what is the PROBABILITY that the shirt is black?

$\frac{3}{23}$

2) If a black shirt was removed from the closet and not replaced, what is the PROBABILITY of then randomly selecting a green shirt?

3) If a black shirt and a green shirt were removed from the closet and not replaced, what is the PROBABILITY of then randomly selecting a white shirt?

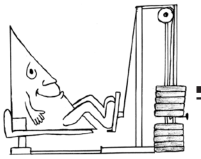
4) What is the PROBABILITY of randomly selecting a red shirt from the closet?

A refrigerator contains 10 bottles of water, 7 bottles of apple juice, and 13 bottles of orange juice.

5) If a drink is randomly selected from the refrigerator, what is the PROBABILITY that the drink is a bottle of water?

6) If a bottle of water was removed from the refrigerator and not replaced, what is the PROBABILITY of then randomly selecting a bottle of orange juice?

7) If a bottle of water and a bottle of orange juice were removed from the refrigerator and not replaced, what is the PROBABILITY of then randomly selecting a drink that is *not* a bottle of water?



• Mastery Check: Probability •

A bag contains 2 red marbles, 4 white marbles, and 6 blue marbles.

1) If a marble is randomly selected from the bag, what is the **PROBABILITY** that the marble is *not* white?

____

2) If a white marble was removed from the bag and not replaced, what is the **PROBABILITY** of then randomly selecting a red marble?

____

A box contains 17 carrots, 18 cabbages, 13 bell peppers, and 4 onions.

3) What is the **PROBABILITY** of randomly selecting a peanut from the basket?

____

4) If a vegetable is randomly selected from the box, what is the **PROBABILITY** that the vegetable is an onion?

____

5) If an onion was removed from the box and not replaced, what is the **PROBABILITY** of then randomly selecting a vegetable that is *not* a carrot?

____

Challenge:

A box contains 12 bananas, 24 apples, 6 oranges, 17 mangos, 5 lemons, and 13 pears.

6) If a fruit is randomly selected from the box, what is the **PROBABILITY** that the fruit is *neither* a banana nor a pear?

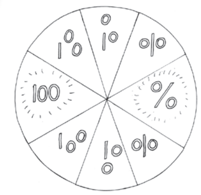
____

• Percent – For Each 100 •

Percent

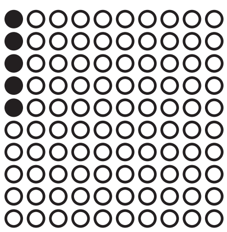
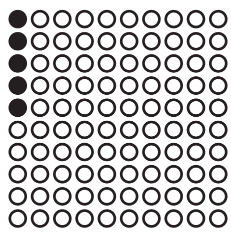
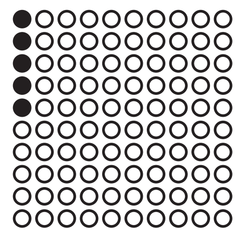
Per means "***for each***" and **cent** means "***100***."
So **percent** means "***for each 100***."

EXAMPLE: **Find 5% of 300.**



5% means "5 for each 100." **So count...**

5 for the *first* 100, **5** for the *second* 100, ***and*** **5** for the *third* 100.

5 + 5 + 5 = 15

So, **5% of 300 is 15**.

Try these:

1) 5% of 200 = ______________ **2)** 9% of 300 = ______________

3) 15% of 200 = ______________ **4)** 15% of 400 = ______________

5) 6% of 200 = ______________ **6)** 6% of 100 = ______________

7) 16% of 200 = ______________ **8)** 4% of 300 = ______________

9) 10% of 200 = ______________ **10)** 8% of 500 = ______________

11) 60% of 300 = ______________ **12)** 10% of 400 = ______________

13) 8% of 300 = ______________ **14)** 8% of 700 = ______________

15) 7% of 300 = ______________ **16)** 7% of 900 = ______________

• Percent – For Each 100 •

1) 11% of 100 = ______________	**2)** 34% of 100 = ______________
3) 8% of 200 = ______________	**4)** 16% of 600 = ______________
5) 23% of 300 = ______________	**6)** 25% of 700 = ______________
7) 12% of 500 = ______________	**8)** 16% of 400 = ______________
9) 9% of 600 = ______________	**10)** 12% of 300 = ______________
11) 16% of 300 = ______________	**12)** 41% of 200 = ______________
13) 13% of 200 = ______________	**14)** 6% of 800 = ______________
15) 9% of 100 = ______________	**16)** 10% of 400 = ______________
17) 15% of 400 = ______________	**18)** 12% of 50 = ______________

---

19) A cell phone costs $100. The sales tax rate is 8%. How much tax is added?

____

20) A ski lift carried 500 riders in one day. 65% of the riders were snowboarders. How many riders were ***not*** snowboarders?

____

21) A sandwich shop sells 200 sandwiches in one day. 45% of the sandwiches are made with wheat bread. How many sandwiches are made with wheat bread?

____

22) An ice cream truck sells 300 ice cream sandwiches in one week. 24% of the ice cream sandwiches have strawberry-flavored ice cream. How many strawberry-flavored ice cream sandwiches are sold?

____

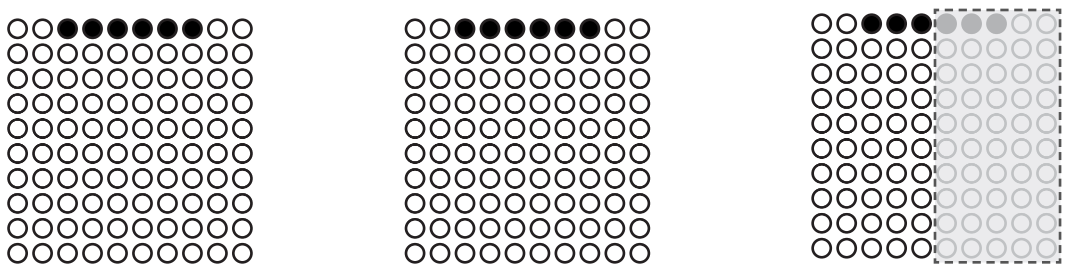
• Percent – For Each 100 •

Direct Teaching

EXAMPLE: **Find 6% of 250.**

6% means "6 for each 100." **So count...**

6 for the *first* 100, **6** for the *second* 100, ***and*** **half of 6** for half of the *third* 100.



6 + 6 + 3 = 15

So, **6% of 250 is 15**.

Try these:

1) 8% of 100 = ______________

2) 8% of 50 = ______________

3) 12% of 150 = ______________

4) 2% of 250 = ______________

5) 13% of 200 = ______________

6) 6% of 150 = ______________

7) 20% of 250 = ______________

8) 10% of 200 = ______________

9) 14% of 150 = ______________

10) 40% of 350 = ______________

11) 21% of 300 = ______________

12) 26% of 400 = ______________

13) 60% of 350 = ______________

14) 12% of 650 = ______________

15) 16% of 550 = ______________

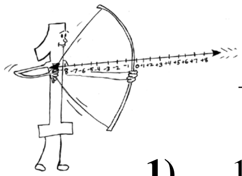
16) 10% of 250 = ______________

17) 8% of 400 = ______________

18) 9% of 300 = ______________

19) 30% of 350 = ______________

20) 7% of 250 = ______________



• Percent – For Each 100 •

1) 1% of 100 = ______________

2) 3% of 150 = ______________

3) 14% of 50 = ______________

4) 10% of 200 = ______________

5) 8% of 150 = ______________

6) 5% of 250 = ______________

7) 16% of 200 = ______________

8) 9% of 300 = ______________

9) 6% of 250 = ______________

10) 11% of 250 = ______________

11) 12% of 150 = ______________

12) 1% of 150 = ______________

13) 16% of 50 = ______________

14) 18% of 2500 = ______________

15) 22% of 300 = ______________

16) 30% of 250 = ______________

17) 14% of 450 = ______________

18) 4% of 50 = ______________

---

19) A ticket to a professional soccer game is $50. The sales tax rate is 9%. How much sales tax is added?

____

20) A roller coaster carried 150 riders in one hour. 26% of the riders were taller than 6 feet. How many riders were taller than 6 feet?

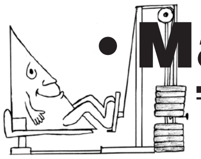
____

21) A diner sold 350 hamburgers in one week. 30% of the hamburgers were served with fries. How many hamburgers were served with fries?

____

22) A neighborhood has 250 families. 34% of the families have two or more children. How many families have ***fewer*** than two children?

____



• Mastery Check: Percent – For Each 100 •

1) 7% of 300 = ______________ **2)** 8% of 250 = ______________

3) 6% of 50 = ______________ **4)** 24% of 200 = ______________

5) 16% of 150 = ______________ **6)** 20% of 150 = ______________

7) 35% of 200 = ______________ **8)** 17% of 300 = ______________

9) 14% of 250 = ______________ **10)** 12% of 350 = ______________

11) 9% of 150 = ______________ **12)** 4% of 150 = ______________

---

13) An apple orchard has 200 trees. 48% of the trees are ready to be picked. How many apple trees are ready to be picked?

____

14) A candy store has 250 varieties of candy. 36% of the varieties contain chocolate. How many varieties of candy contain chocolate?

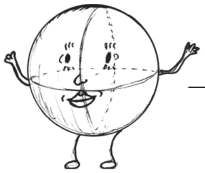
____

15) An aquarium contains 150 exhibits. 12% of the exhibits are closed due to maintenance. How many exhibits are ***open***?

____

Challenge:

16) $8\frac{1}{2}$% of 400 = ______________ **17)** $6\frac{1}{2}$% of 250 = ______________



• Portioning – Unequal Amounts • ☆

Direct Teaching

Let's take a look at a "**portioning**" word problem.

EXAMPLE: Mr. Swain pays his sons, Tim and Ben, \$80 to paint two rooms of his house. Tim works for 5 hours and Ben works for 3 hours. How should they divide the money so that each person gets a fair share?

Steps to Solve:

STEP 1:

Find the total number of *equal parts* into which the whole is divided.

5 hours + **3** hours = **8** hours

So there are **8** total *equal parts*.

STEP 2:

Find the *value* of each part (hour) by dividing the whole (\$80) by the total number of parts (8).

\$80 (whole) ÷ **8** (parts) =

\$10 for each part

Tim					Ben		
10	10	10	10	10	10	10	10

80

STEP 3:

Find each person's *fair share* of the earnings.

TIM: **5** hours × **\$10** = \$50

BEN: **3** hours × **\$10** = \$30

***Try this*:**

1) Lauren and Josh share a car. Lauren uses 7 gallons of gas every week, while Josh uses 5 gallons. Together, they pay \$36 for gas. How should they divide the cost of the gas so that they each pay a fair share?

LAUREN = ____________ **JOSH** = ____________

• Portioning – Unequal Amounts •

Draw a Picture

1) Max and Nick decide to buy and share a new tablet. Max will use the tablet for 6 hours per week, while Nick will use the tablet for 3 hours per week. The tablet costs $270. How should they divide the cost so that each person pays a fair share?

MAX = ____________ NICK = ____________

---

2) Cristina and Domenico take turns walking their neighbor's dogs. Cristina walks them 5 times and Domenico walks them 2 times. They are paid $35 for their services. How should they divide the money so that each person gets a fair share?

CRISTINA = ____________ DOMENICO = ____________

---

3) Michelle and Alex recycled their bottles and cans together and received $44 in compensation. Michelle contributed 4 pounds of recyclables and Alex contributed 7 pounds. How should they divide the money so that each person gets a fair share?

MICHELLE = ____________ ALEX = ____________

---

4) Edward and Brittany went to the market and bought two dozen eggs. Edward paid $3 and Brittany paid $5. How should they divide the eggs so that each person gets a fair amount?

EDWARD = ____________ BRITTANY = ____________

• Portioning – Hidden Ratios •

Direct Teaching

Some **portioning** word problems will not give the exact amount of parts. Instead, they may give a ratio in the form of “**twice as many**” or “**three times as many**.”

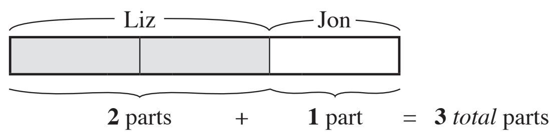
Example: At the train show, Liz took twice as many photos as Jon did. Together, they took 51 photos. How many photos did they each take?

Steps to Solve:

Step 1:

Find the total number of *equal parts* into which the whole is divided.

“**Twice as many**” means **2** *for every* **1**.

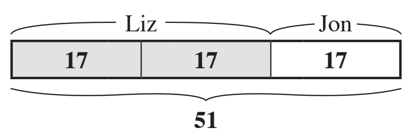


Step 2:

Find the *value* of each part by dividing the whole (51) by the total number of parts (3).

51 (whole) ÷ **3** (parts) =

17 photos for each part



Step 3:

Find the number of photos that each person took.

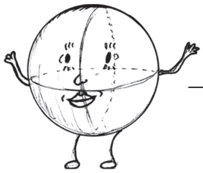
Liz: **2** parts × **17** = 34 photos

Jon: **1** part × **17** = 17 photos

***Try this*:**

1) Albert and Beth collected shells at the beach. Albert collected three times as many shells as Beth. Together, they gathered 44 shells. How many shells did they collect individually?

Albert = ______________ **Beth** = ______________



• Portioning – Hidden Ratios •

Draw a Picture

1) Chris and Holly take turns mowing their neighbor's lawn. Chris works for twice as many hours as Holly. They are paid a total of $60. How should they divide the money so that each person gets a fair share?

Chris = ____________ **Holly =** ____________

---

2) Marla has three times as much money as Billy. Together, they have $280. How much money does each person have?

Marla = ____________ **Billy =** ____________

---

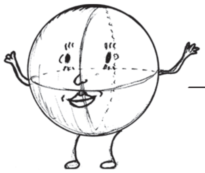
3) In a basketball game, the winning team scored twice as many points as the losing team. Together, they scored a total of 81 points. How many points did each team score?

Winning Team = ____________ **Losing Team =** ____________

---

4) Ray bought three times as many tickets as Christine. Together, they have 76 tickets. How many tickets did they each buy?

Ray = ____________ **Christine =** ____________



• Portioning •

1) Jerry, Charlie, and Maya buy a bag of 32 potatoes to share. Jerry pays $6, Charlie pays $7, and Maya pays $3. How should they divide the potatoes so that each person gets a fair share?

Jerry = ____________ **Charlie** = ____________ **Maya** = ____________

---

2) Bill drank twice as much tomato juice as Eric. Together, they drank 48 ounces of tomato juice. How many ounces of tomato juice did they each drink?

Bill = ____________ **Eric** = ____________

---

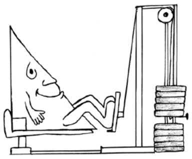
3) The winning baseball team scored five times as many runs as the losing team. Together, they scored a total of 18 runs. How many runs did each team score?

Winning Team = ____________ **Losing Team** = ____________

---

4) Sam and Jin share a car. Sam uses 14 gallons of gas per week, while Jin only uses 8 gallons. Together, they pay $66 for gas each week. How should they divide the cost of the gas so that each person pays a fair share?

Sam = ____________ **Jin** = ____________



• Mastery Check: Portioning •

1) Martha and George take turns babysitting. Martha works for 4 hours and George works for 2 hours. They are paid a total of $24. How should they divide the money so that each person gets a fair share?

MARTHA = ____________ GEORGE = ____________

---

2) Austin has twice as much money as Jay. Together, they have $150. How much money does each person have?

AUSTIN = ____________ JAY = ____________

---

3) Peter and David bought a package of toilet paper that contained 63 rolls. Peter paid $12 and David paid $9. How should they divide up the rolls so that each person gets a fair share?

PETER = ____________ DAVID = ____________

Challenge:

4) In a football game, the losing team scored half as many points as the winning team. Together, they scored a total of 63 points. How many points did each team score?

WINNING TEAM = ____________ LOSING TEAM = ____________

• Reasoning in Groups •

3 tennis balls are in 1 container.

1) How many containers are needed for 6 tennis balls? ____________

2) How many tennis balls can 4 containers hold? ____________

3) How many containers are needed for 9 tennis balls? ____________

4) How many tennis balls can 6 containers hold? ____________

6 bananas are in 1 sack.

5) How many sacks are needed for 18 bananas? ____________

6) How many bananas are in 7 sacks? ___________

7) How many sacks are needed for 30 bananas? ____________

8) How many bananas are in 10 sacks? ____________

• Reasoning in Groups •

Proportional Thinking

If 2 apples cost 5 quarters...

2 apples | **5 quarters**

1) How many *quarters* do **4 apples** cost?

APPLES	QUARTERS
2	5
2	5

____

2) How many *apples* can you buy with **15 quarters**?

APPLES	QUARTERS
	5
	5
	5

____

3) How many *quarters* do **8 apples** cost?

APPLES	QUARTERS

____

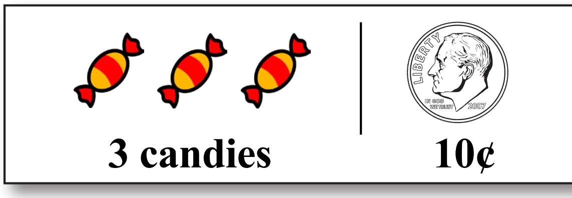
4) How many *apples* can you buy with **25 quarters**?

APPLES	QUARTERS

____

• Reasoning in Groups •

If 3 candies cost 10¢...



1) How much do 12 candies cost?

How many *groups* of **3** candies are there *inside of* **12** candies? [4]

10¢, [4] times = ________
COST OF 12 CANDIES

2) How many candies can you buy for 60¢?

How many *groups* of **10¢** are there *inside of* **60¢**? [ ]

3 candies, [ ] times = ________
NUMBER OF CANDIES FOR 60¢

3) How much do 24 candies cost?

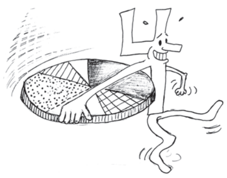
How many *groups* of **3** candies are there *inside of* **24** candies? [ ]

10¢, [ ] times = ________
COST OF 24 CANDIES

4) How many candies can you buy for $1.00?

How many *groups* of **10¢** are there *inside of* **$1.00**? [ ]

3 candies, [ ] times = ________
NUMBER OF CANDIES FOR $1.00



• Reasoning in Groups •

Proportional Thinking

If a 6-foot man casts a 9-foot shadow...

1) How tall is a tree that casts a 45-foot shadow?

How many *groups* of **9** feet are there *inside of* **45** feet? $\boxed{}$

6 feet, $\boxed{}$ times = ______

HEIGHT OF THE TREE (IN FEET)

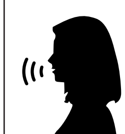
---

2) How long is a tree's shadow if the tree is 18 feet tall?

How many *groups* of **6** feet are there *inside of* **18** feet? $\boxed{}$

9 feet, $\boxed{}$ times = ______

LENGTH OF THE TREE'S SHADOW (IN FEET)



Direct Teaching

3) How tall is a tree that casts an **81-foot shadow**?

6 feet, $\boxed{}$ times = ______

HEIGHT OF THE TREE (IN FEET)

---

4) How long is a tree's shadow if the **tree** is **24 feet** tall?

9 feet, $\boxed{}$ times = ______

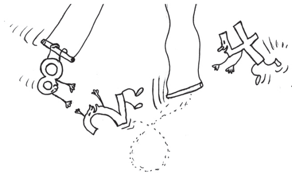
LENGTH OF THE TREE'S SHADOW (IN FEET)

---

5) How tall is a tree that casts a **72-foot shadow**?

6 feet, $\boxed{}$ times = ______

HEIGHT OF THE TREE (IN FEET)



• Reasoning in Groups •

If 4 pounds of grapes costs $7.00...

1) How much does **12 pounds of grapes** cost? ________________

2) How many pounds of grapes can you buy for **$35.00**? ________________

3) How much does **28 pounds of grapes** cost? ________________

4) How many pounds of grapes can you buy for **$84.00**? ________________

5) How much does **40 pounds of grapes** cost? ________________

6) How many pounds of grapes can you buy for **$56.00**? ________________

If 2 pieces of gum cost 25¢...

7) How much do **6 pieces of gum** cost? ________________

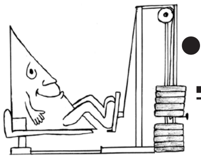
8) How many pieces of gum can you buy for **$2.00**? ________________

9) How much do **10 pieces of gum** cost? ________________

10) How many pieces of gum can you buy for **$2.50**? ________________

11) How much do **24 pieces of gum** cost? ________________

12) How many pieces of gum can you buy for **$3.25**? ________________



• Mastery Check: Reasoning in Groups •

If 3 candies cost 25¢...

1) How many candies can you buy for **$2.00**? ________________

2) How much do **18 candies** cost? ________________

If a 6-foot man casts an 8-foot shadow...

3) How tall is a tree that casts a **24-foot shadow**? ________________

4) How long is a tree's shadow if the **tree** is **42 feet** tall? ________________

If 8 cookies cost $3.00...

5) How many cookies can you buy for **$27.00**? ________________

6) How much do **40 cookies** cost? ________________

Challenge:

If a 4-foot child casts a 6-foot shadow...

7) How tall is a tree that casts a **9-foot shadow**? ________________

8) How tall is a tree that casts a **15-foot shadow**? ________________

• Reasoning in Groups •

If 2 cookies cost $3.00...

2 cookies **$3.00**

---

1) How many cookies can you buy for $18.00?

How many *groups* of **$3.00** are there *inside of* **$18.00**? [6]

2 cookies, [6] times = ________
NUMBER OF COOKIES FOR $18.00

---

2) How much do 16 cookies cost?

How many *groups* of **2** cookies are there *inside of* **16** cookies? [ ]

$3.00, [ ] times = ________
COST OF 16 COOKIES

---

3) How many cookies can you buy for $27.00?

How many *groups* of **$3.00** are there *inside of* **$27.00**? [ ]

2 cookies, [ ] times = ________
NUMBER OF COOKIES FOR $27.00

---

4) How much do 22 cookies cost?

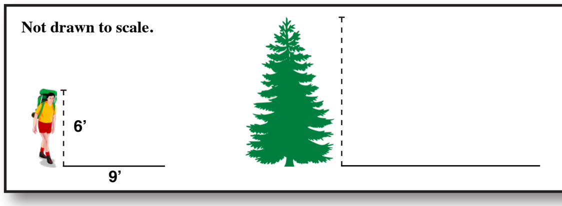
How many *groups* of **2** cookies are there *inside of* **22** cookies? [ ]

$3.00, [ ] times = ________
COST OF 22 COOKIES

• Proportional Reasoning •

Proportional Thinking

If a 6–foot man casts a 9–foot shadow...



1) How tall is a tree that casts a 45–foot shadow?

How many *groups* of **9** feet are there *inside of* **45** feet? $\boxed{5}$

6 feet, $\boxed{5}$ times = ______

HEIGHT OF THE TREE (IN FEET)

2) How long is a tree's shadow if the tree is 18 feet tall?

How many *groups* of **6** feet are there *inside of* **18** feet? $\boxed{}$

9 feet, $\boxed{}$ times = ______

LENGTH OF THE TREE'S SHADOW (IN FEET)

3) How tall is a tree that casts an **81–foot shadow**?

Direct Teaching

6 feet, $\boxed{}$ times = ______

HEIGHT OF THE TREE (IN FEET)

4) How long is a tree's shadow if the **tree** is **24 feet** tall?

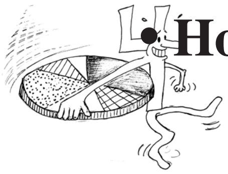
9 feet, $\boxed{}$ times = ______

LENGTH OF THE TREE'S SHADOW (IN FEET)

5) How tall is a tree that casts a **72–foot shadow**?

6 feet, $\boxed{}$ times = ______

HEIGHT OF THE TREE (IN FEET)



• How Many of "These" Are There Inside of "That"? • ☆

Wholes and Parts

1) How many *whole* **10s** are there inside of **55**? _______

What *fractional part* of **10** is left over? _______

So, *exactly* how many **10s** are there inside of **55**? _________

---

2) How many *whole* **8s** are there inside of **20**? _______

What *fractional part* of **8** is left over? _______

So, *exactly* how many **8s** are there inside of **20**? _________

---

3) How many *whole* **3s** are there inside of $\mathbf{4\frac{1}{2}}$? _______

What *fractional part* of **3** is left over? _______

So, ***exactly*** how many **3s** are there inside of $\mathbf{4\frac{1}{2}}$? _________

---

4) ***Exactly*** how many **6s** are there inside of **21**? _________

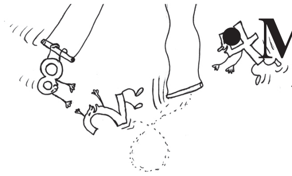
5) ***Exactly*** how many **5s** are there inside of $\mathbf{32\frac{1}{2}}$? _________

6) ***Exactly*** how many **2s** are there inside of **19**? _________

7) ***Exactly*** how many **7s** are there inside of $\mathbf{17\frac{1}{2}}$? _________

8) ***Exactly*** how many **12s** are there inside of **66**? _________

9) ***Exactly*** how many **4s** are there inside of **18**? _________



Multiplying by a **Whole–and–a–Half** •

☆

Mental Math

1) **6**, $1\frac{1}{2}$ *times* is...

______ + ______ = ______

6, 1 *time* | half of **6** | 6, $1\frac{1}{2}$ *times*

2) **4**, $2\frac{1}{2}$ *times* is...

______ + ______ = ______

4, 2 *times* | half of **4** | 4, $2\frac{1}{2}$ *times*

3) **3**, $4\frac{1}{2}$ *times* is...

______ + ______ = ______

3, 4 *times* | half of **3** | 3, $4\frac{1}{2}$ *times*

4) **8**, $3\frac{1}{2}$ *times* is...

______ + ______ = ______

8, 3 *times* | half of **8** | 8, $3\frac{1}{2}$ *times*

---

5) **7**, $1\frac{1}{2}$ *times* is ________.

6) **10**, $5\frac{1}{2}$ *times* is ________.

7) **9**, $2\frac{1}{2}$ *times* is ________.

8) **4**, $4\frac{1}{2}$ *times* is ________.

9) **12**, $5\frac{1}{2}$ *times* is ________.

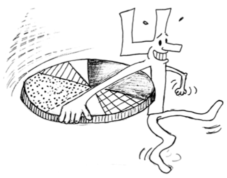
10) **11**, $6\frac{1}{2}$ *times* is ________.

11) **5**, $3\frac{1}{2}$ *times* is ________.

12) **8**, $2\frac{1}{2}$ *times* is ________.

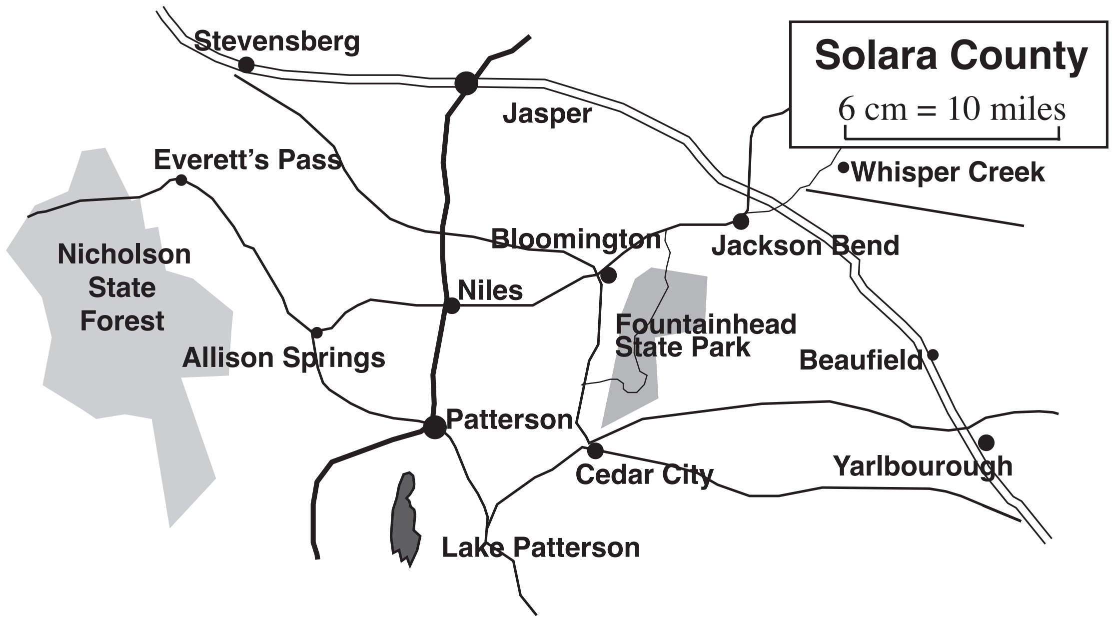
13) **6**, $5\frac{1}{2}$ *times* is ________.

14) **9**, $4\frac{1}{2}$ *times* is ________.



• Proportional Reasoning • ☆

In a scale drawing, 6 centimeters represent 10 miles.



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1) How many centimeters represent 25 miles?

Exactly how many *groups* of **10 miles** are there *inside of* **25 miles**? $\boxed{2\frac{1}{2}}$

6 cm, $\boxed{2\frac{1}{2}}$ times = __________ **cm**

---

2) How many miles are represented by 9 cm?

Exactly how many *groups* of **6 cm** are there *inside of* **9 cm**? ☐

10 miles, ☐ times = __________ **miles**

---

3) How many centimeters represent 45 miles?

Exactly how many *groups* of **10 miles** are there *inside of* **45 miles**? ☐

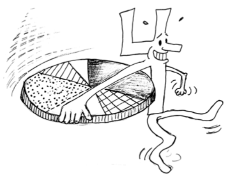
6 cm, ☐ times = __________ **cm**

---

4) How many miles are represented by 21 cm?

Exactly how many *groups* of **6 cm** are there *inside of* **21 cm**? ☐

10 miles, ☐ times = __________ **miles**



• Proportional Reasoning •

☆

Proportional Thinking

On a map, 3 inches represent 10 miles.

---

1) How many inches represent **25** miles?

3 inches, ☐ times = __________ inches

---

2) How many miles are represented by $\mathbf{4\frac{1}{2}}$ inches?

10 miles, ☐ times = __________ miles

Label your answers.

3) How many inches represent **35 miles**? ______________

4) How many miles are represented by $\mathbf{16\frac{1}{2}}$ **inches**? ______________

5) How many inches represent **45 miles**? ______________

6) How many miles are represented by **24 inches**? ______________

7) How many inches represent **65 miles**? ______________

8) How many miles are represented by $\mathbf{31\frac{1}{2}}$ **inches**? ______________

• Proportional Reasoning •

Proportional Thinking

If a 5–foot woman casts an 8–foot shadow...

1) How tall is a tree that casts a **28–foot shadow**?

5 feet, $\boxed{}$ times = __________ feet

2) How long is a tree's shadow if the **tree** is **35 feet** tall?

8 feet, $\boxed{}$ times = __________ feet

Label your answers.

3) How tall is a tree that casts a **44–foot shadow**? ______________

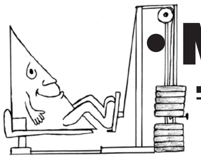
4) How long is a tree's shadow if the **tree** is **$12\frac{1}{2}$ feet** tall? ______________

5) How tall is a tree that casts an **80–foot shadow**? ______________

6) How long is a tree's shadow if the **tree** is **$7\frac{1}{2}$ feet** tall? ______________

7) How tall is a tree that casts a **36–foot shadow**? ______________

8) How long is a tree's shadow if the **tree** is **45 feet** tall? ______________



• Mastery Check: Proportional Reasoning •

☆

If a 6–foot man casts an 8–foot shadow...

1) How tall is a tree that casts a **32–foot shadow**? ________________

2) How long is a tree's shadow if the **tree** is **9 feet** tall? ________________

3) How tall is a tree that casts a **20–foot shadow**? ________________

4) How long is a tree's shadow if the **tree** is **48 feet** tall? ________________

5) How tall is a tree that casts a **44–foot shadow**? ________________

In a scale drawing, 3 inches represent 10 miles.

6) How many inches represent **90 miles**? ______________

7) How many miles are represented by **36 inches**? ______________

8) How many inches represent **35 miles**? ______________

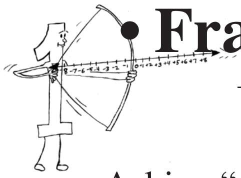
9) How many miles are represented by **$7\frac{1}{2}$ inches**? ______________

10) How many inches represent **45 miles**? ______________

Challenge:

If a 10–foot pole casts a 13–foot shadow...

11) How long is a pole's shadow if the **pole** is **$12\frac{1}{2}$ feet** tall? ________________



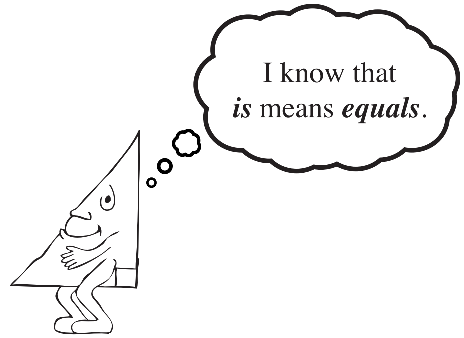
• Fractional Parts – Finding the *What* PART •

Direct Teaching

Asking "*what part* of a bigger number is a smaller number?" is the same as asking "*what fraction* of the bigger number is the smaller number?"

EXAMPLE: What part of **48** is **36**?

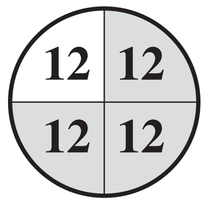
STEP 1: Identify the **WHOLE**. 48 (The **WHOLE**)

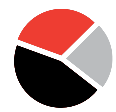


STEP 2: Compare the **GIVEN PART** to the **WHOLE** as a fraction in lowest terms.

$$\frac{36}{48} = \frac{3}{4}$$

The **GIVEN PART** (36) / The **WHOLE** (48) = The *What* Part ($\frac{3}{4}$)





Wholes and Parts

So, $\frac{3}{4}$ of 48 is 36.
What Part

Try these:

1) What part of **21** is **14**?

STEP 1: ______ The **WHOLE**

STEP 2: The **GIVEN PART** / The **WHOLE** = The *What* Part (in *lowest terms*)

So, ______ of 21 is 14.
What Part

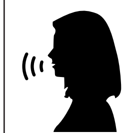
2) What part of **80** is **24**?

STEP 1: ______ The **WHOLE**

STEP 2: The **GIVEN PART** / The **WHOLE** = The *What* Part (in *lowest terms*)

So, ______ of 80 is 24.
What Part

• Fractional Parts – Finding the *What* Part •



Direct Teaching

1) What part of **44** is **32**?

Step 1:

The **Whole**

Step 2:

The **Given Part**

The **Whole**

=

The *What* Part (in *lowest terms*)

So, ______ of 44 is 32.

What Part

2) **40** is what part of **50**?

Step 1:

The **Whole**

Step 2:

The **Given Part**

The **Whole**

=

The *What* Part (in *lowest terms*)

So, 40 is ______ of 50.

What Part

3) What part of **72** is **45**?

The **Given Part**

The **Whole**

=

The *What* Part (in *lowest terms*)

So, ______ of 72 is 45.

What Part

4) **11** is what part of **66**?

The **Given Part**

The **Whole**

=

The *What* Part (in *lowest terms*)

So, 11 is ______ of 66.

What Part

5) What part of **36** is **21**?

The **Given Part**

The **Whole**

=

The *What* Part (in *lowest terms*)

So, ______ of 36 is 21.

What Part

6) **18** is what part of **24**?

The **Given Part**

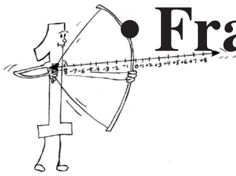
The **Whole**

=

The *What* Part (in *lowest terms*)

So, 18 is ______ of 24.

What Part



• Fractional Parts – Finding the *What* Part •

☆

1) What part of **14** is **12**?

The Given Part / The Whole = The *What* Part (in *lowest terms*)

So, ______ of 14 is 12.
What Part

2) **32** is what part of **72**?

The Given Part / The Whole = The *What* Part (in *lowest terms*)

So, 32 is ______ of 72.
What Part

3) **63** is what part of **70**?

The Given Part / The Whole = The *What* Part (in *lowest terms*)

So, 63 is ______ of 70.
What Part

4) What part of **60** is **24**?

The Given Part / The Whole = The *What* Part (in *lowest terms*)

So, ______ of 60 is 24.
What Part

Write all answers as fractions in lowest terms.

5) What part of 88 is 33? ________

6) 7 is what part of 28? ________

7) 6 is what part of 9? ________

8) What part of 48 is 20? ________

9) What part of 36 is 30? ________

10) 100 is what part of 110? ________

• Fractional Parts – Finding the *What* PART •

Write all answers as fractions in lowest terms.

1) What part of 40 is 35? _______

2) 12 is what part of 16? _______

3) What part of 30 is 15? _______

4) 50 is what part of 70? _______

5) What part of 108 is 24? _______

6) 33 is what part of 55? _______

7) What part of 20 is 14? _______

8) 28 is what part of 77? _______

9) What part of 96 is 88? _______

10) 9 is what part of 54? _______

A class has 8 boys and 10 girls.

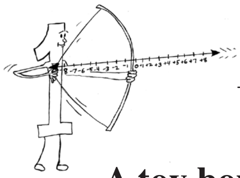
11) What part of the class are girls? ________

12) What part of the class are boys? ________

A candy jar contains 9 chocolates, 14 gumballs, and 7 mints.

13) What part of the candies are chocolates? ________

14) What part of the candies are gumballs? ________



• Fractional Parts •

A toy box contains 16 action figures, 4 stuffed animals, and 12 toy cars.

1) What part of the toys are stuffed animals? ________

2) What part of the toys are toy cars? ________

3) What part of the toys are action figures? ________

A refrigerator contains 6 juice boxes, 9 water bottles, and 12 soda cans.

4) What part of the drinks are soda cans? ________

5) What part of the drinks are juice boxes? ________

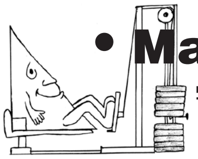
6) What part of the drinks are water bottles? ________

A piggy bank contains 6 pennies, 19 nickels, 10 dimes, and 7 quarters.

7) What part of the coins are dimes? ________

8) What part of the coins are pennies? ________

9) What part of the coins are quarters? ________



• Mastery Check: Fractional Parts – Finding the *What* Part •

1) What part of 12 is 9? ________

2) 36 is what part of 81? ________

3) What part of 56 is 40? ________

4) 28 is what part of 32? ________

5) What part of 20 is 15? ________

6) 12 is what part of 18? ________

7) What part of 121 is 66? ________

8) 18 is what part of 30? ________

A bag contains 3 red marbles, 4 white marbles, and 5 blue marbles.

9) What part of the marbles are blue? _________

10) What part of the marbles are red? _________

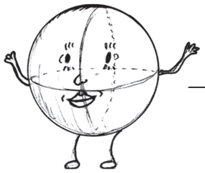
A florist is selling 4 roses, 10 tulips, and 6 orchids.

11) What part of the flowers are orchids? _________

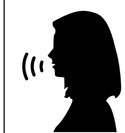
12) What part of the flowers are roses? _________

Challenge:

13) What part of 1,000 is 375? _________



• SAMEness •

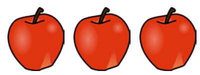
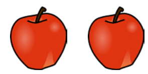
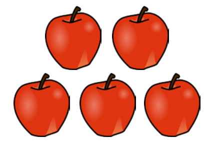


Direct Teaching

The Law of SAMEness

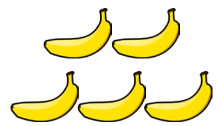
We can only add and subtract things that are of the ***same denomination***, things that have the ***same name***.

EXAMPLE 1:

 +  = 

3 apples	+	2 apples	=	5 apples

EXAMPLE 2:

 +  = 

3 bananas	+	2 bananas	=	5 bananas

Try these:

1) **8** apples + **3** apples = **11**________________

2) **5** cups + **4** cups = ______ cups

3) **9** nickels – **2** nickels = **7**________________

4) **6** dogs + **5** dogs = **11** ________________

5) **11** bananas – **8** bananas = ______ bananas

6) **3** tenths + **5** tenths = ______ tenths

7) **12** sixths – **7** sixths = **5**________________

8) **2** fourths + **1** fourth = **3**________________

• SAMEness •

1) **2** fifths + **2** fifths = ______ fifths

2) **4** sevenths + **1** seventh = **5**________________

3) **10** elevenths – **3** elevenths = ______ elevenths

4) **3** sixths + **2** sixths = ______ sixths

5) **9** ninths – **5** ninths = **4**________________

6) **5** eighths + **2** eighths = **7**________________

---

7) $\frac{3}{7} + \frac{1}{7} = \frac{\square}{\mathbf{7}}$

8) $\frac{9}{10} - \frac{3}{10} = \frac{\square}{\mathbf{10}}$

9) $\frac{1}{5} + \frac{4}{5} = \frac{\mathbf{5}}{\square}$

10) $\frac{5}{12} + \frac{5}{12} = \frac{\mathbf{10}}{\square}$

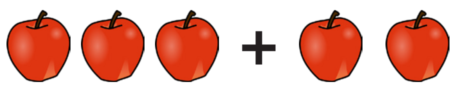
11) $\frac{7}{11} - \frac{3}{11} = \frac{\square}{\square}$

12) $\frac{1}{8} + \frac{4}{8} = \frac{\square}{\square}$

• SAMEness •

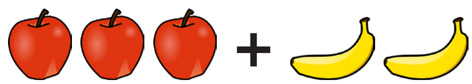
The Law of SAMEness

We can only add and subtract things that are of the ***same denomination***, things that have the ***same name***.



3 apples + 2 apples = 5 apples

3 bananas + 2 bananas = 5 bananas



3 apples + 2 bananas = 5____________ (*not* banapples!)

Direct Teaching

Try these:

1) **3** dogs + **5** cats = **8**________________

2) **6** dimes + **9** nickels = **15**________________

3) **4** carrots + **8** peas = **12**________________

4) **10** fruit flies + **7** mosquitoes = **17**________________

5) **1** foot – **3** inches = **9**________________

6) **2** hours + **8** minutes = **128**________________

7) **3** weeks – **4** days = **17**________________

8) **5** pennies + **2** quarters = **55**________________

• SAMEness •

The Law of SAMEness

1) **6** phones + **7** phones = **13**________________

2) **15** letters – **8** letters = **7**________________

3) **10** inches + **2** feet = **34**________________

4) **3** dimes + **4** nickels = **50**________________

5) **$2\frac{1}{2}$** hours – **30** minutes = **2**________________

6) **2** weeks + **5** days = **19**________________

7) **4** yards + **24** inches = **14**________________

8) **3** fourths + **1** half = **5**________________

9) **5** eighths – **1** half = **1**________________

10) **1** sixth + **1** half = **4**________________

---

11) Explain why **3** apples *plus* **2** bananas does **NOT** equal 5 ***banapples***.

____

____

____

12) Explain why **2** feet *minus* **6** inches equals 18 inches.

____

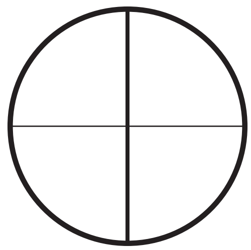
____

____

• Adding Fractions •

The Law of SAMEness

1) Shade in $\frac{1}{2}$ of the circle.

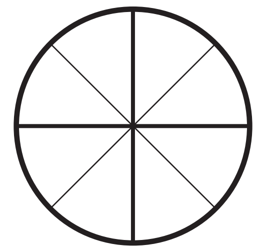


Now shade in $\frac{1}{4}$ more.

How much of the circle is shaded? ______

$$\frac{1}{2} + \frac{1}{4} = \_\_\_\_\_\_$$

2) Shade in $\frac{3}{4}$ of the circle.

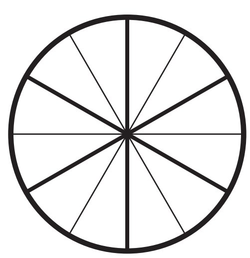


Now shade in $\frac{1}{8}$ more.

How much of the circle is shaded? ______

$$\frac{3}{4} + \frac{1}{8} = \_\_\_\_\_\_$$

3) Shade in $\frac{1}{6}$ of the circle.

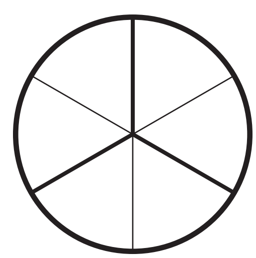


Now shade in $\frac{3}{12}$ more.

How much of the circle is shaded? ______

$$\frac{1}{6} + \frac{3}{12} = \_\_\_\_\_\_$$

4) Shade in $\frac{1}{3}$ of the circle.



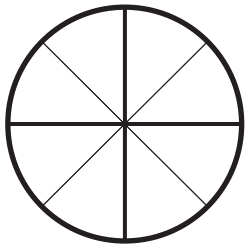
Now shade in $\frac{1}{2}$ more.

How much of the circle is shaded? ______

$$\frac{1}{3} + \frac{1}{2} = \_\_\_\_\_\_$$

• Adding Fractions •

1) Shade in $\frac{1}{4}$ of the circle.

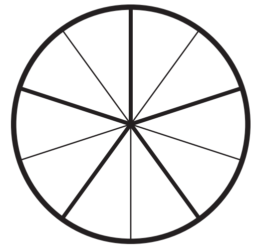


Now shade in $\frac{3}{8}$ more.

How much of the circle is shaded? ______

$$\frac{1}{4} + \frac{3}{8} = \_\_\_\_\_\_$$

2) Shade in $\frac{1}{5}$ of the circle.

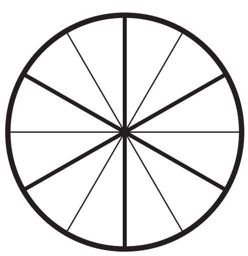


Now shade in $\frac{5}{10}$ more.

How much of the circle is shaded? ______

$$\frac{1}{5} + \frac{5}{10} = \_\_\_\_\_\_$$

3) Shade in $\frac{3}{4}$ of the circle.

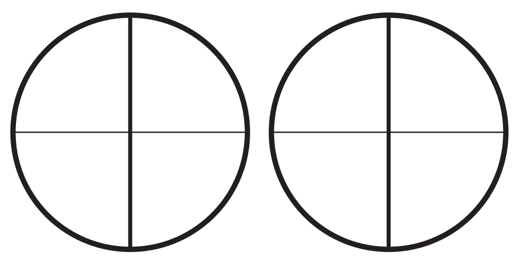


Now shade in $\frac{1}{6}$ more.

How much of the circle is shaded? ______

$$\frac{3}{4} + \frac{1}{6} = \_\_\_\_\_\_$$

4) Shade in $\frac{1}{2}$ of a circle.



Now shade in $\frac{3}{4}$ more.

How much of the circles are shaded? ______

$$\frac{1}{2} + \frac{3}{4} = \_\_\_\_\_\_ = \_\_\_\_\_\_$$

ANOTHER NAME

Direct Teaching

• Adding and Subtracting Fractions •

Direct Teaching

To add and subtract fractions with *different names* (denominators), we *rename* the fractions using a name that they have ***in common*** before solving.

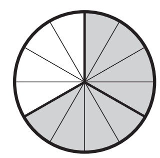
EXAMPLE: $\frac{2}{3} + \frac{1}{4} = ?$

THINK: What is the *smallest number* that both **3** and **4** go into evenly? $\boxed{12}$ COMMON NAME

Rename each fraction (if necessary) using the **COMMON NAME**. Then solve.

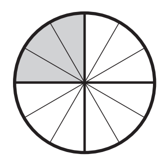
$$\frac{2}{3} \overset{\times 4}{\underset{\times 4}{=}} \frac{8}{\boxed{12}}$$

COMMON NAME

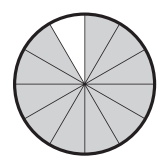


$$+\ \frac{1}{4} \overset{\times 3}{\underset{\times 3}{=}} \frac{3}{\boxed{12}}$$

COMMON NAME



$$\frac{\boxed{11}}{\boxed{12}}$$



Try these: **To make the COMMON NAME (denominator), find the smallest number that both denominators go into evenly.**

1)

$$\frac{4}{9} = \frac{}{\boxed{}}$$

COMMON NAME

$$+\ \frac{2}{5} = \frac{}{\boxed{}}$$

COMMON NAME

$$\frac{\boxed{}}{\boxed{}}$$

2)

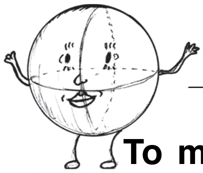
$$\frac{5}{6} = \frac{}{\boxed{}}$$

COMMON NAME

$$-\ \frac{3}{8} = \frac{}{\boxed{}}$$

COMMON NAME

$$\frac{\boxed{}}{\boxed{}}$$



• Adding and Subtracting Fractions •

☆

The Law of SAMEness

To make the COMMON NAME (denominator), find the smallest number that both denominators go into evenly.

1)

$$\frac{3}{5} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$+\ \frac{1}{7} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$\frac{\boxed{}}{\boxed{}}$$

2)

$$\frac{4}{7} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$-\ \frac{3}{8} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$\frac{\boxed{}}{\boxed{}}$$

3)

$$\frac{7}{12} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$-\ \frac{4}{9} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$\frac{\boxed{}}{\boxed{}}$$

4)

$$\frac{5}{6} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$+\ \frac{1}{9} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$\frac{\boxed{}}{\boxed{}}$$

5)

$$\frac{1}{5} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$+\ \frac{2}{3} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$\frac{\boxed{}}{\boxed{}}$$

6)

$$\frac{9}{10} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$-\ \frac{7}{8} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$\frac{\boxed{}}{\boxed{}}$$

• Adding and Subtracting Fractions •

To make the COMMON NAME (denominator), find the smallest number that both denominators go into evenly.

1)

$$\frac{2}{7} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$+\ \frac{5}{8} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$\frac{\boxed{}}{\boxed{}}$$

2)

$$\frac{6}{11} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$-\ \frac{1}{3} = \frac{\quad}{\boxed{}}$$

COMMON NAME

$$\frac{\boxed{}}{\boxed{}}$$

Write answers as proper fractions or mixed numbers in lowest terms.

3) $\frac{1}{8} + \frac{11}{12}$

4) $\frac{8}{9} - \frac{1}{2}$

5) $\frac{7}{10} + \frac{4}{15}$

6) $\frac{3}{4} + \frac{3}{7}$

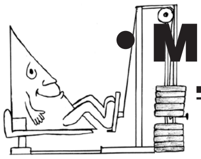
7) $\frac{2}{3} - \frac{1}{6}$

8) $\frac{5}{6} + \frac{2}{11}$

9) $\frac{11}{15} - \frac{1}{4} =$ ______

10) $\frac{4}{9} + \frac{5}{36} =$ ______

11) $\frac{2}{7} - \frac{1}{6} =$ ______



• Mastery Check: Adding and Subtracting Fractions •

Write answers as proper fractions or mixed numbers in lowest terms.

1) $\begin{array}{r} \frac{2}{3} \\ + \quad \frac{1}{4} \\ \hline \end{array}$

2) $\begin{array}{r} \frac{1}{5} \\ + \quad \frac{5}{7} \\ \hline \end{array}$

3) $\begin{array}{r} \frac{3}{8} \\ - \quad \frac{1}{6} \\ \hline \end{array}$

4) $\begin{array}{r} \frac{5}{6} \\ + \quad \frac{2}{9} \\ \hline \end{array}$

5) $\begin{array}{r} \frac{1}{3} \\ - \quad \frac{1}{6} \\ \hline \end{array}$

6) $\begin{array}{r} \frac{7}{12} \\ + \quad \frac{3}{10} \\ \hline \end{array}$

---

7) $\frac{3}{4} + \frac{1}{2} =$ ______

8) $\frac{5}{11} + \frac{4}{7} =$ ______

9) $\frac{2}{3} - \frac{3}{8} =$ ______

10) $\frac{13}{16} - \frac{1}{4} =$ ______

11) $\frac{14}{15} - \frac{5}{6} =$ ______

12) $\frac{3}{4} + \frac{2}{3} =$ ______

Challenge:

13) $\frac{1}{2} - \frac{1}{3} - \frac{1}{6} =$ ______

14) $\begin{array}{r} 4\frac{1}{6} \\ + \quad 5\frac{3}{7} \\ \hline \end{array}$

• Subtracting Mixed Numbers with Like Denominators •

Direct Teaching

EXAMPLE:

$$\begin{array}{r} 5\frac{8}{9} \\ -\ 1\frac{5}{9} \\ \hline ? \end{array}$$

Steps to Solve:

STEP 1:

Subtract the fractions.

$$\begin{array}{r} 5\frac{8}{9} \\ -\ 1\frac{5}{9} \\ \hline \frac{3}{9} \end{array}$$

STEP 2:

Subtract the whole numbers.

$$\begin{array}{r} 5\frac{8}{9} \\ -\ 1\frac{5}{9} \\ \hline 4\frac{3}{9} \end{array}$$

STEP 3:

Write the answer in simplified form.

$$4\frac{3}{9} = \mathbf{4\frac{1}{3}}$$

Try these: **All answers should be written in simplified form.**

1) $\begin{array}{r} 8\frac{5}{6} \\ -\ 6\frac{4}{6} \\ \hline \end{array}$

2) $\begin{array}{r} 7\frac{10}{11} \\ -\ 4\frac{3}{11} \\ \hline \end{array}$

3) $\begin{array}{r} 9\frac{25}{27} \\ -\ 2\frac{7}{27} \\ \hline \end{array}$

Subtraction with Borrowing – Whole minus Mixed Numbers

We can subtract **MIXED NUMBERS** from **WHOLE NUMBERS** *mentally* or by using the following steps:

EXAMPLE:

$$\begin{array}{r} 7 \\ -\ 3\frac{2}{10} \\ \hline ? \end{array}$$

Steps to Solve:

STEP 1:

Borrow ***one whole*** (1) from the whole number (7) and rename it using the denominator from the fraction in the second number (10ths).

$$\begin{array}{r} \overset{6}{\cancel{7}}\frac{10}{10} \\ -\ 3\frac{2}{10} \\ \hline \end{array}$$

STEP 2:

Subtract the fractions, and then subtract the whole numbers.

$$\begin{array}{r} \overset{6}{\cancel{7}}\frac{10}{10} \\ -\ 3\frac{2}{10} \\ \hline 3\frac{8}{10} \end{array}$$

STEP 3:

Write the answer in simplified form.

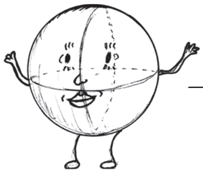
$$3\frac{8}{10} = \mathbf{3\frac{4}{5}}$$

Try these: **All answers should be written in simplified form.**

1) $\begin{array}{r} 4 \\ -\ 1\frac{5}{9} \\ \hline \end{array}$

2) $\begin{array}{r} 8 \\ -\ 3\frac{2}{5} \\ \hline \end{array}$

3) $\begin{array}{r} 12 \\ -\ 6\frac{8}{14} \\ \hline \end{array}$



• Working with Unity •

In each exercise, fill in the box to solve.

1) $5 = 4 + \frac{\boxed{3}}{3}$

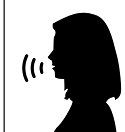
2) $7 = 6 + \frac{\boxed{}}{7}$

3) $10 = 9 + \frac{\boxed{}}{14}$

4) $16 = 15 + \frac{\boxed{}}{11}$

5) $3 = 2 + \frac{\boxed{}}{13}$

6) $13 = 12 + \frac{\boxed{}}{26}$



Direct Teaching

7) $4\frac{7}{8} = 3\frac{8}{8} + \frac{7}{8} = 3\frac{\boxed{15}}{8}$

8) $2\frac{4}{5} = 1\frac{5}{5} + \frac{4}{5} = 1\frac{\boxed{}}{5}$

9) $9\frac{5}{12} = 8\frac{12}{12} + \frac{5}{12} = 8\frac{\boxed{}}{12}$

10) $16\frac{9}{21} = 15\frac{21}{21} + \frac{9}{21} = 15\frac{\boxed{}}{21}$

---

11) $8\frac{5}{6} = 7\frac{\boxed{11}}{6}$

12) $11\frac{1}{2} = 10\frac{\boxed{}}{2}$

13) $5\frac{12}{16} = 4\frac{\boxed{}}{16}$

14) $12\frac{3}{4} = 11\frac{\boxed{}}{4}$

15) $17\frac{9}{10} = 16\frac{\boxed{}}{10}$

16) $15\frac{13}{22} = 14\frac{\boxed{}}{22}$

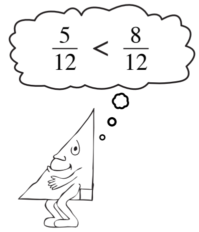
17) $10\frac{11}{20} = 9\frac{\boxed{}}{20}$

18) $14\frac{7}{9} = 13\frac{\boxed{}}{9}$

19) $6\frac{18}{25} = 5\frac{\boxed{}}{25}$

EXAMPLE:

$$\begin{array}{r} 6\frac{5}{12} \\ -\ 2\frac{8}{12} \\ \hline ? \end{array}$$



When the fractional part of the first number is smaller than the fractional part of the second number, we can use the following steps:

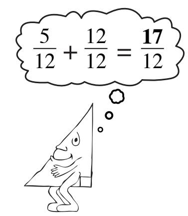
Steps to Solve:

STEP 1:

In the first number, borrow ***one whole*** $\left(\frac{12}{12}\right)$ from the whole number (6) and add it to the fraction $\left(\frac{5}{12}\right)$.

This makes the fraction in the first number bigger than the fraction in the second number.

$$\begin{array}{r} \overset{5}{\cancel{6}}\,\dfrac{\overset{\boxed{17}}{\cancel{5}}}{12} \\ -\ 2\dfrac{8}{12} \\ \hline \end{array}$$



STEP 2:

Subtract the fractions, and then subtract the whole numbers.

$$\begin{array}{r} \overset{5}{\cancel{6}}\,\dfrac{\overset{\boxed{17}}{\cancel{5}}}{12} \\ -\ 2\dfrac{8}{12} \\ \hline 3\dfrac{9}{12} \end{array}$$

STEP 3:

Write the answer in simplified form.

$$3\frac{9}{12} = \mathbf{3\frac{3}{4}}$$

Try these: **All answers should be written in simplified form.**

1) $\begin{array}{r} \cancel{9}\,\dfrac{\cancel{4}}{15} \\ -\ 3\dfrac{7}{15} \\ \hline \end{array}$

2) $\begin{array}{r} \cancel{4}\,\dfrac{\cancel{5}}{9} \\ -\ 2\dfrac{6}{9} \\ \hline \end{array}$

3) $\begin{array}{r} 1\cancel{1}\,\dfrac{\cancel{3}}{18} \\ -\ 7\dfrac{10}{18} \\ \hline \end{array}$

• Subtracting Mixed Numbers with Unlike Denominators • ☆

Example: $5\frac{3}{8} - 2\frac{9}{10} = \textbf{?}$

Direct Teaching

When subtracting mixed numbers whose fractions have *different names* (denominators), we can use the following steps to solve.

Steps to Solve:

Step 1:

Rename the fractions using a COMMON NAME (denominator).

Note: Finding the *smallest* COMMON NAME (the ***least common denominator***) will lead to the ***least amount of reducing*** in the answer.

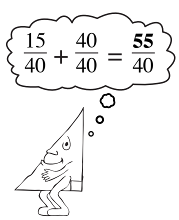
The Law of SAMEness

$$\begin{array}{rl} & 5\frac{3}{8} = 5\frac{15}{\underset{\text{COMMON NAME}}{\boxed{40}}} \\ - & 2\frac{9}{10} = 2\frac{36}{\underset{\text{COMMON NAME}}{\boxed{40}}} \\ \hline \end{array}$$

Step 2:

In the first number, borrow ***one whole*** from the whole number if it is necessary in order to subtract the fractions.

$$\begin{array}{rr} & \overset{4}{\cancel{5}}\frac{\overset{\boxed{55}}{\cancel{15}}}{40} \\ - & 2\frac{36}{40} \\ \hline \end{array}$$



Step 3:

Subtract (and ***reduce*** if possible).

$$\begin{array}{rr} & \overset{4}{\cancel{5}}\frac{\overset{\boxed{55}}{\cancel{15}}}{40} \\ - & 2\frac{36}{40} \\ \hline & 2\frac{19}{40} \end{array}$$

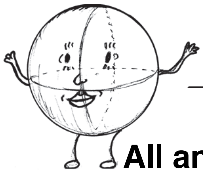
So, $5\frac{3}{8} - 2\frac{9}{10} = \mathbf{2\frac{19}{40}}$.

Try these: **All answers should be written in simplified form.**

1) $\begin{array}{rr} & 7\,\frac{2}{3} \\ - & 1\,\frac{4}{5} \\ \hline \end{array}$

2) $\begin{array}{rr} & 10\,\frac{1}{8} \\ - & 5\,\frac{5}{6} \\ \hline \end{array}$

3) $\begin{array}{rr} & 12\,\frac{3}{4} \\ - & 9\,\frac{11}{12} \\ \hline \end{array}$



• Subtracting **Mixed Numbers** •

All answers should be written in simplified form.

1) $$\begin{array}{r} 14\frac{3}{7} \\ -\ 7\frac{10}{49} \\ \hline \end{array}$$

2) $$\begin{array}{r} 10\frac{2}{3} \\ -\ 2\frac{17}{18} \\ \hline \end{array}$$

3) $$\begin{array}{r} 16\frac{5}{6} \\ -\ 4\frac{3}{5} \\ \hline \end{array}$$

4) $$\begin{array}{r} 4\frac{1}{3} \\ -\ 3\frac{9}{25} \\ \hline \end{array}$$

5) $$\begin{array}{r} 9\frac{1}{2} \\ -\ 5\frac{4}{5} \\ \hline \end{array}$$

6) $$\begin{array}{r} 6\frac{7}{12} \\ -\ 1\frac{8}{9} \\ \hline \end{array}$$

7) $$\begin{array}{r} 12\frac{2}{3} \\ -\ 8\frac{11}{20} \\ \hline \end{array}$$

8) $$\begin{array}{r} 17\frac{3}{5} \\ -\ 8\frac{7}{11} \\ \hline \end{array}$$

9) $$\begin{array}{r} 15\frac{4}{7} \\ -\ 9\frac{9}{10} \\ \hline \end{array}$$

---

10) $7\frac{1}{9} - 4\frac{6}{7} =$ ______

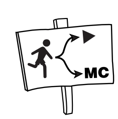
11) $8\frac{7}{10} - 3\frac{1}{4} =$ ______

12) $13\frac{1}{2} - 5\frac{8}{11} =$ ______

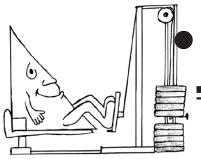
13) $11\frac{21}{64} - 2\frac{7}{8} =$ ______

14) $12\frac{2}{3} - 8\frac{14}{17} =$ ______

15) $17\frac{5}{8} - 6\frac{7}{12} =$ ______



Bypass



• Mastery Check: Subtracting Mixed Numbers •

☆

All answers should be written in simplified form.

1) $$\begin{array}{r} 6\frac{3}{5} \\ -\ 2\frac{5}{6} \\ \hline \end{array}$$

2) $$\begin{array}{r} 9\frac{2}{9} \\ -\ 4\frac{8}{27} \\ \hline \end{array}$$

3) $$\begin{array}{r} 5\frac{1}{7} \\ -\ 3\frac{3}{4} \\ \hline \end{array}$$

4) $$\begin{array}{r} 8\frac{4}{5} \\ -\ 1\frac{7}{11} \\ \hline \end{array}$$

5) $$\begin{array}{r} 10\frac{1}{6} \\ -\ 7\frac{5}{8} \\ \hline \end{array}$$

6) $$\begin{array}{r} 11\frac{1}{2} \\ -\ 5\frac{16}{25} \\ \hline \end{array}$$

7) $$\begin{array}{r} 14\frac{2}{7} \\ -\ 4\frac{5}{6} \\ \hline \end{array}$$

8) $$\begin{array}{r} 17\frac{13}{16} \\ -\ 9\frac{1}{4} \\ \hline \end{array}$$

9) $$\begin{array}{r} 13\frac{7}{12} \\ -\ 6\frac{8}{9} \\ \hline \end{array}$$

Challenge:

10) $$\begin{array}{r} 45\frac{9}{16} \\ -\ 13\frac{49}{72} \\ \hline \end{array}$$

11) $10\frac{1}{1000} - 6\frac{1}{100} =$ ________

• Multiplying with **Mixed Numbers** •

Direct Teaching

To multiply with MIXED NUMBERS, convert the MIXED NUMBERS to IMPROPER FRACTIONS and then ***multiply***.

EXAMPLE: $$2\frac{5}{8} \times 3\frac{1}{7} = ?$$

First convert the MIXED NUMBERS to IMPROPER FRACTIONS.

$$\frac{21}{8} \times \frac{22}{7}$$

Then ***multiply***.

$$\frac{{}^{3}\cancel{21}}{{}_{4}\cancel{8}} \times \frac{\cancel{22}^{11}}{\cancel{7}_{1}} = \frac{33}{4} = \underline{8\frac{1}{4}}$$

Try these:

Convert any mixed numbers to improper fractions and then multiply.

Write answers in lowest terms, and convert any improper fraction answers to mixed numbers.

1) $2\frac{1}{4} \times 4\frac{2}{3} =$ ______ **2)** $4\frac{2}{5} \times 1\frac{11}{14} =$ ______

3) $2\frac{4}{5} \times \frac{1}{7} =$ ______ **4)** $\frac{5}{18} \times 8 =$ ______

5) $9\frac{1}{3} \times 3\frac{6}{7} =$ ______ **6)** $1\frac{1}{6} \times 7\frac{1}{2} =$ ______

7) $6 \times 5\frac{1}{4} =$ ______ **8)** $\frac{11}{20} \times 2\frac{2}{5} =$ ______

• Multiplying with **Mixed Numbers** •

Write answers in lowest terms and convert any improper fraction answers to mixed numbers.

1) $9\frac{1}{3} \times \frac{7}{8} =$ ______

2) $1\frac{3}{10} \times 2\frac{6}{7} =$ ______

3) $4\frac{7}{12} \times 3\frac{3}{11} =$ ______

4) $2\frac{1}{6} \times \frac{9}{11} =$ ______

5) $8\frac{4}{7} \times 1\frac{1}{4} =$ ______

6) $\frac{5}{8} \times 4\frac{2}{3} =$ ______

7) $9\frac{3}{4} \times 2\frac{8}{13} =$ ______

8) $\frac{8}{9} \times 5\frac{1}{4} =$ ______

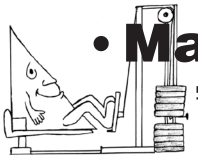
9) $3\frac{5}{9} \times 3\frac{3}{8} =$ ______

10) $6 \times 1\frac{13}{21} =$ ______

11) $1\frac{1}{10} \times 4\frac{3}{8} =$ ______

12) $\frac{19}{20} \times 4 =$ ______

Bypass



• Mastery Check: Multiplying with Mixed Numbers • ☆

Write answers in lowest terms and convert any improper fraction answers to mixed numbers.

1) $2\frac{2}{5} \times \frac{7}{8} =$ ______

2) $4\frac{2}{3} \times 1\frac{3}{4} =$ ______

3) $1\frac{5}{8} \times 2\frac{2}{11} =$ ______

4) $6 \times 1\frac{1}{14} =$ ______

5) $6\frac{1}{4} \times 8\frac{4}{5} =$ ______

6) $2\frac{2}{9} \times \frac{18}{25} =$ ______

7) $\frac{9}{16} \times 10 =$ ______

8) $3\frac{3}{7} \times 8\frac{1}{6} =$ ______

Challenge:

9) $\frac{3}{5} \times 8\frac{1}{3} \times 3\frac{2}{3} =$ ______

10) $2\frac{29}{35} \times \frac{10}{18} =$ ______

• Dividing with **Mixed Numbers** •

To divide with MIXED NUMBERS, convert the MIXED NUMBERS to IMPROPER FRACTIONS and then follow the procedure for ***dividing proper fractions***.

EXAMPLE:

$$3\tfrac{3}{7} \div 1\tfrac{7}{9} = ?$$

First convert the MIXED NUMBERS to IMPROPER FRACTIONS.

$$\frac{24}{7} \div \frac{16}{9}$$

Then ***multiply*** $\frac{24}{7}$ by the **reciprocal** of $\frac{16}{9}$ to find the answer. Check if any factors can be cross canceled.

$$\frac{\overset{3}{\cancel{24}}}{7} \times \frac{9}{\underset{2}{\cancel{16}}} = \frac{27}{14} = \underline{1\tfrac{13}{14}}$$

Try these:

Convert any mixed numbers to improper fractions and then divide.

Write answers in lowest terms, and convert any improper fraction answers to mixed numbers.

1) $2\frac{2}{5} \div 1\frac{5}{13} =$ ______

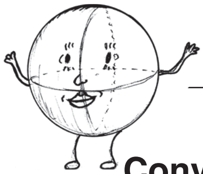
2) $3\frac{6}{7} \div 2\frac{11}{14} =$ ______

3) $12 \div 1\frac{3}{11} =$ ______

4) $1\frac{7}{8} \div \frac{9}{14} =$ ______

5) $4\frac{4}{5} \div 18 =$ ______

6) $9\frac{1}{6} \div 1\frac{11}{24} =$ ______



• Dividing with **Mixed Numbers** •

Convert any mixed numbers to improper fractions and then divide.
Write answers in lowest terms, and convert any improper fraction answers to mixed numbers.

1) $3\frac{3}{8} \div 1\frac{7}{11} =$ ______

2) $16 \div 3\frac{7}{15} =$ ______

3) $2\frac{6}{13} \div 3 =$ ______

4) $3\frac{5}{7} \div \frac{11}{14} =$ ______

5) $\frac{20}{27} \div 1\frac{7}{9} =$ ______

6) $2\frac{4}{17} \div 2\frac{12}{13} =$ ______

7) $3\frac{5}{9} \div 2\frac{2}{21} =$ ______

8) $5\frac{5}{8} \div 18 =$ ______

9) $3\frac{3}{5} \div 4\frac{8}{15} =$ ______

10) $10\frac{5}{12} \div 15 =$ ______

• Dividing with **Mixed Numbers** •

Write answers in lowest terms and convert any improper fraction answers to mixed numbers.

1) $2 \div 3\frac{2}{11} =$ ______

2) $6\frac{8}{11} \div 8 =$ ______

3) $10\frac{1}{3} \div 1\frac{1}{6} =$ ______

4) $4\frac{7}{12} \div 4\frac{3}{8} =$ ______

5) $\frac{15}{16} \div 1\frac{13}{14} =$ ______

6) $2\frac{1}{12} \div \frac{13}{15} =$ ______

7) $4\frac{4}{9} \div 12 =$ ______

8) $1\frac{5}{18} \div 7\frac{2}{3} =$ ______

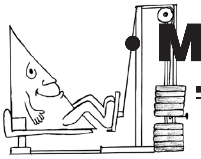
9) $5\frac{1}{4} \div \frac{9}{10} =$ ______

10) $10\frac{6}{7} \div 4 =$ ______

11) $7\frac{7}{10} \div 1\frac{5}{16} =$ ______

12) $2\frac{4}{13} \div 3\frac{3}{7} =$ ______

Bypass



• Mastery Check: Dividing with Mixed Numbers •

Write answers in lowest terms and convert any improper fraction answers to mixed numbers.

1) $2\frac{7}{10} \div 6 =$ ______

2) $3\frac{1}{4} \div 1\frac{9}{16} =$ ______

3) $2\frac{14}{15} \div 10\frac{1}{12} =$ ______

4) $\frac{15}{17} \div 1\frac{3}{7} =$ ______

5) $4\frac{2}{7} \div 3\frac{3}{14} =$ ______

6) $20 \div 3\frac{7}{11} =$ ______

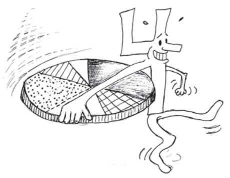
7) $3\frac{11}{13} \div 4 =$ ______

8) $1\frac{17}{18} \div \frac{14}{27} =$ ______

Challenge:

9) $2\frac{1}{3} \times 4\frac{5}{6} \div 1\frac{1}{6} =$ ______

10) $4\frac{10}{15} \div 1\frac{10}{14} =$ ______



• Adding Decimal Fractions •

The Law of SAMEness

Example: **Line up the numbers by place value. Then, solve.**

How much is 63.5 + 1.74?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
			1			
		6	3	.	5	0
+			1	.	7	4
		6	5	.	2	4

Reminder:
The decimal point separates the wholes (65) from the parts (24 hundredths).

Try these: **Line up the numbers by place value. Then, solve.**

1) How much is 14.7 + 3.69?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
+				.		

2) How much is 123.86 + 37.4?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
+				.		

• Adding **Decimal Fractions** •

Line up the numbers by place value. Then, solve.

1) How much is 14.92 + 0.7?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
+				.		

2) How much is 32.4 + 6.89?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
+				.		

Line up the numbers by place value. Then, solve.

3) 57.6 + 8.42 = __________

4) 3.95 + 1.2 = __________

5) 21.68 + 3.37 = __________

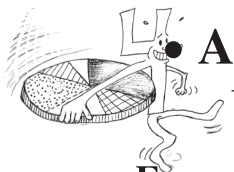
6) 5.4 + 73.8 = __________

7) 8.29 + 7.06 = __________

8) 3.51 + 8.9 = __________

9) 31.6 + 14.56 = __________

10) 19.86 + 87.11 = __________



• Adding Whole Numbers and **Decimal Fractions** •

The Law of SAMEness

EXAMPLE: **Line up the numbers by place value. Then, solve.**

How much is **14.62 + 9**?

Note that "**9**" is a **WHOLE NUMBER** that can also be written as **9.0**, **9.00**, **9.000**, etc.

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
		1				
		1	4	.	6	2
+			9	.	0	0
		2	3	.	6	2

Notice that zeros are written into the *tenths* and *hundredths* places, since "9" has zero *tenths* and zero *hundredths*.

Try these: **Line up the numbers by place value. Then, solve.**

1) How much is 6.84 + 2?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
+				.		

2) How much is 7 + 16.48?

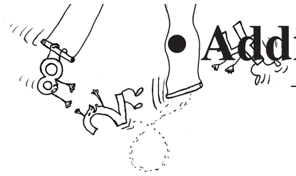
	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
+				.		

3) How much is 9.6 + 38?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
+				.		

4) How much is 105 + 7.25?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
+				.		



• Adding Whole Numbers and Decimal Fractions •

☆

Line up the numbers by place value. Then, solve.

1) How much is 6.01 + 19?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
+				.		

2) How much is 54 + 8.93?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
+				.		

Line up the numbers by place value. Then, solve.

3) 10 + 4.6 = __________

4) 8.26 + 5 = __________

5) 46.1 + 3 = __________

6) 12 + 7.11 = __________

7) 0.9 + 21 = __________

8) 3.45 + 8 = __________

9) 7 + 3.4 = __________

10) 11.4 + 5 = __________

• Adding Decimal Fractions •

Solve.

1) $23 + 8.09 =$ __________

2) $14.2 + 5.88 =$ __________

3) $7.45 + 3.65 =$ __________

4) $0.02 + 9.9 =$ __________

5) $32.71 + 5.6 + 4 =$ __________

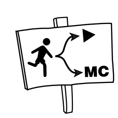
6) $7.6 + 8 + 4.53 =$ __________

7) $3 + 4.01 + 0.8 =$ __________

8) $9.5 + 22 + 5.74 =$ __________

9) Andy bought a movie ticket for $12, a hotdog for $3.75, and a soda for $1.50. How much did he spend altogether?

____



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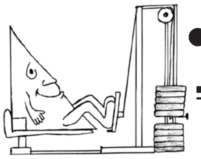


• Mastery Check: Adding Decimals •

Solve.

1) 3.14 + 0.6 = __________

2) 4.23 + 7.9 = __________

3) 14 + 0.62 = __________

4) 5.3 + 22 = __________

5) 9.3 + 6.02 = __________

6) 8.09 + 14.47 = __________

7) 15.81 + 2.7 + 3 = __________

8) 19.69 + 2 + 4.7 = __________

Challenge:

9) 569 + 0.008 + 42.01 + 12 = ______________

• Subtracting **Decimal Fractions** •

☆

The Law of SAMEness

Example: How much is 63.5 – 1.74?

Line up the numbers by place value. Then, solve.

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
			2		14	
		6	~~3~~	.	~~5~~	10
–			1	.	7	4
		6	1	.	7	6

"Borrow" when needed. Here, we borrow from the *tenths* first. Then, we borrow from the ones.

Reminder:
A decimal point is needed in the answer to separate the wholes from the parts (between the *ones* and the *tenths*).

Try these: **Line up the numbers by place value. Then, solve.**

1) How much is 37.4 – 0.65?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
–				.		

2) How much is 21.36 – 5.4?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
–				.		

• Subtracting Decimal Fractions •

The Law of SAMEness

Line up the numbers by place value. Then, solve.

1) How much is 19.5 – 8.67?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
–				.		

2) How much is 53.96 – 25.67?

	100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
				.		
–				.		

Line up the numbers by place value. Then, solve.

3) 10.5 – 4.6 = __________

4) 8.26 – 5.3 = __________

5) 46.1 – 6.4 = __________

6) 12.87 – 3.79 = __________

7) 6.7 – 0.58 = __________

8) 28.45 – 8.4 = __________

9) 57.65 – 8.39 = __________

10) 11.45 – 5.36 = __________

• Subtracting Decimal Fractions •

Show your work neatly.

1) $8.53 - 4.87 =$ __________

2) $13.59 - 2.5 =$ __________

3) $36.48 - 4.6 =$ __________

4) $27.4 - 9.62 =$ __________

5) $54.48 - 0.35 =$ __________

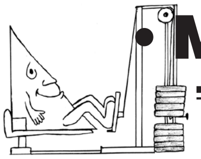
6) $10.2 - 2.89 =$ __________

7) $9.96 - 3.359 =$ __________

8) $125.57 - 35.18 =$ __________

9) Sylvia had $12.35 before she bought lunch for $5.78. How much does she have left?

____



• Mastery Check: Subtracting Decimals • ☆

Show your work neatly.

1) $42.24 - 0.1 =$ __________

2) $18.29 - 3.7 =$ __________

3) $24.76 - 4.89 =$ __________

4) $31.6 - 9.53 =$ __________

5) $63.16 - 0.29 =$ __________

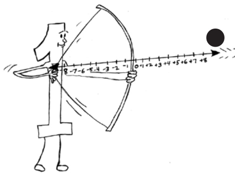
6) $19.37 - 8.6 =$ __________

7) $52.9 - 4.89 =$ __________

8) $71.11 - 0.33 =$ __________

Challenge:

9) $4{,}783.039 - 179.472 =$ __________



• Estimating the Product of Decimals •

☆

In each exercise, estimate the answer by rounding each number to the nearest WHOLE NUMBER.

1) $4.94 \times 5.8 = ?$

$\underline{\;5\;} \times \underline{\;6\;} = \boxed{}$ ESTIMATE

2) $3.11 \times 6.72 = ?$

$\underline{\quad} \times \underline{\quad} = \boxed{}$ ESTIMATE

3) $2.18 \times 8.1 = ?$

$\underline{\quad} \times \underline{\quad} = \boxed{}$ ESTIMATE

4) $22.6 \times 5.76 = ?$

$\underline{\quad} \times \underline{\quad} = \boxed{}$ ESTIMATE

5) $35.6 \times 4.09 = ?$

$\underline{\quad} \times \underline{\quad} = \boxed{}$ ESTIMATE

6) $9.3 \times 10.85 = ?$

$\underline{\quad} \times \underline{\quad} = \boxed{}$ ESTIMATE

7) $6.61 \times 33.3 = ?$

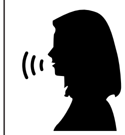
$\underline{\quad} \times \underline{\quad} = \boxed{}$ ESTIMATE

8) $18.84 \times 1.89 = ?$

$\underline{\quad} \times \underline{\quad} = \boxed{}$ ESTIMATE

• Multiplying Decimals •

☆



Direct Teaching

$$3.95 \times 2.1 = ?$$

Steps to Solve:

Step 1:

Estimate by rounding each number to the nearest **WHOLE NUMBER.**

$3.95 \times 2.1 = ?$

$\underline{4} \times \underline{2} = \boxed{8}$

Estimate

Step 2:

Line up the numbers by the last digit in each number, **not** by place value.

$$\begin{array}{r} 3.95 \\ \times \quad 2.1 \\ \hline \end{array}$$

Step 3:

Multiply normally, ignoring the decimal points.

$$\begin{array}{r} 3.95 \\ \times \quad 2.1 \\ \hline 395 \\ + \quad 7900 \\ \hline 8295 \end{array}$$

Step 4:

Find the total number of decimal places in the multiplied numbers.

$$\begin{array}{r} 3.95 \\ \times \quad 2.1 \\ \hline 8295 \end{array}$$

3 TOTAL DECIMAL PLACES

Step 5:

In your answer, move the decimal point to the *left* by the total number of decimal places.

$$\begin{array}{r} 3.95 \\ \times \quad 2.1 \\ \hline 8.295 \end{array}$$

3 decimal places

Step 6:

Check for reasonableness. The estimate was 8, so 8.295 is very reasonable.

Estimate = 8

Actual Answer = **8.295**

Try these:

1) $4.6 \times 3.2 = ?$

_____ × _____ = $\boxed{}$

Estimate

____

Actual Answer

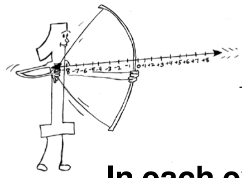
2) $9.18 \times 6.3 = ?$

_____ × _____ = $\boxed{}$

Estimate

____

Actual Answer



• Multiplying Decimals •

In each exercise, estimate the answer by first rounding each number to the nearest WHOLE NUMBER. Then find the actual answer.

1) 4.6 × 3.2 = **?**

5 × 3 = 15

ESTIMATE

______ ACTUAL ANSWER

2) 3.14 × 7.9 = **?**

____ × ____ = ____ ESTIMATE

______ ACTUAL ANSWER

3) 2.05 × 6.3 = **?**

____ × ____ = ____ ESTIMATE

______ ACTUAL ANSWER

4) 11.8 × 9.2 = **?**

____ × ____ = ____ ESTIMATE

______ ACTUAL ANSWER

5) 30.2 × 3.1 = **?**

____ × ____ = ____ ESTIMATE

______ ACTUAL ANSWER

6) 6.86 × 5.91 = **?**

____ × ____ = ____ ESTIMATE

______ ACTUAL ANSWER

• Multiplying Decimals •

1) $4.18 \times 2.4 = ?$

ESTIMATE

ACTUAL ANSWER

2) $1.25 \times 6.3 = ?$

ESTIMATE

ACTUAL ANSWER

3) $12.3 \times 9.7 = ?$

ESTIMATE

ACTUAL ANSWER

4) $10.6 \times 4.02 = ?$

ESTIMATE

ACTUAL ANSWER

5) $$\begin{array}{r} 8.19 \\ \times \quad 4.2 \\ \hline \end{array}$$

6) $$\begin{array}{r} 23.1 \\ \times \quad 0.25 \\ \hline \end{array}$$

7) $$\begin{array}{r} 6.7 \\ \times \quad 5.9 \\ \hline \end{array}$$

8) $$\begin{array}{r} 5.28 \\ \times \quad 7.6 \\ \hline \end{array}$$

9) $$\begin{array}{r} 4.03 \\ \times \quad 3.7 \\ \hline \end{array}$$

10) $$\begin{array}{r} 2.34 \\ \times \quad 1.43 \\ \hline \end{array}$$

• Multiplying Decimals •

1) 0.69 × 4.2 = ________

2) 3.27 × 1.1 = ________

3) 10.63 × 7.9 = ________

4) 8.02 × 5.8 = ________

5)
$$\begin{array}{r} 0.92 \\ \times\quad 4.7 \\ \hline \end{array}$$

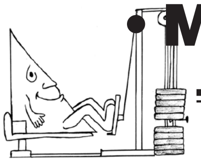
6)
$$\begin{array}{r} 4.1 \\ \times\quad 2.8 \\ \hline \end{array}$$

7)
$$\begin{array}{r} 10.2 \\ \times\quad 1.6 \\ \hline \end{array}$$

8)
$$\begin{array}{r} 13.7 \\ \times\quad 6.4 \\ \hline \end{array}$$

9)
$$\begin{array}{r} 3.29 \\ \times\quad 5.37 \\ \hline \end{array}$$

10)
$$\begin{array}{r} 15.4 \\ \times\quad 2.48 \\ \hline \end{array}$$



• **Mastery Check:** Multiplying Decimals > 1 • ☆

1) $3.42 \times 1.1 =$ ________

2) $2.5 \times 6.3 =$ ________

3) $9.86 \times 4.2 =$ ________

4) $5.03 \times 7.5 =$ ________

5)
$$\begin{array}{r} 4.3 \\ \times \quad 2.8 \\ \hline \end{array}$$

6)
$$\begin{array}{r} 1.29 \\ \times \quad 3.7 \\ \hline \end{array}$$

7)
$$\begin{array}{r} 7.94 \\ \times \quad 8.3 \\ \hline \end{array}$$

Challenge:

8) $0.03 \times 0.02 =$ ________

• Dividing Whole Numbers by **Decimals** •

Direct Teaching

Before dividing by a decimal number, the **DIVISOR** (the "outside" number) needs to be changed to a whole number. Then, we can solve using long division.

EXAMPLE: $0.15\overline{)375}$ = **?**

Steps to Solve:

STEP 1:

Move the decimal point in the **DIVISOR** (0.15) ***two places to the right***.

(because there are two decimal places in the **DIVISOR**)

$0.15.\overline{)375}$

2 places

This is the same as multiplying by 100, which makes the **DIVISOR** a whole number.
(0.15 × 100 = 15)

STEP 2:

Move the decimal point in the **DIVIDEND** (375) ***two places to the right*** and write in zeros to fill in the places correctly.

REMINDER: The decimal point in a whole number is to the right of the *ones* place.

$15.\overline{)375.00.}$

2 places

Since the **DIVISOR** was multiplied by 100, the **DIVIDEND** must also be multiplied by 100.

STEP 3:

Now that the **DIVISOR** (15) is a whole number and the **DIVIDEND** (37,500) has been changed appropriately, we can ***divide***.

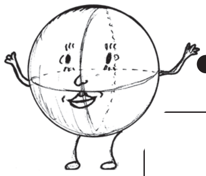
$$
\begin{array}{r}
2{,}500 \\
15.\overline{)\,37{,}500.} \\
\underline{-30} \\
75 \\
\underline{-75} \\
00 \\
\underline{-0} \\
00 \\
\underline{-0} \\
0
\end{array}
$$

Try these:

1) $0.12\overline{)408}$

2) $0.4\overline{)322}$

3) $0.25\overline{)675}$



• Dividing Whole Numbers by **Decimals** • ☆

1) $0.07\overline{)581.00}$

2) $3.9\overline{)156}$

3) $0.26\overline{)91}$

4) $1.8\overline{)765}$

5) $0.11\overline{)869}$

6) $5.4\overline{)351}$

7) $0.42\overline{)315}$

8) $0.3\overline{)84}$

9) $2.7\overline{)1,512}$

• Dividing Whole Numbers by **Decimals** •

1) $4.6\overline{)782.0}$

2) $0.19\overline{)912}$

3) $0.5\overline{)427}$

4) $0.31\overline{)806}$

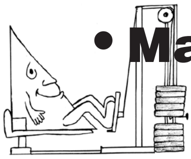
5) $2.3\overline{)69}$

6) $0.64\overline{)768}$

7) $0.02\overline{)14}$

8) $1.7\overline{)1,139}$

9) $0.28\overline{)532}$



• Mastery Check: Dividing Whole Numbers by Decimals •

1) $0.25\overline{)875}$

2) $1.4\overline{)539}$

3) $0.07\overline{)203}$

4) $3.6\overline{)27}$

5) $0.59\overline{)944}$

6) $4.3\overline{)3{,}612}$

Challenge:

7) $0.85\overline{)719{,}695}$

• Converting Fractions to Terminating Decimals • ☆

EXAMPLE: Convert $\frac{3}{25}$ to a decimal.

Since the denominator (25) divides evenly into 100, we can find an equivalent fraction with a denominator of 100 and write it in decimal form.

STEP 1. Find the equivalent fraction with a denominator of 10 or 100.

$$\frac{3}{25} = \frac{\boxed{12}}{100}$$

STEP 2. Write the equivalent fraction in decimal form.

$$\frac{\boxed{12}}{100} = \mathbf{0.12}$$

Try these: **In each exercise, find the equivalent fraction with a denominator of 10 or 100. Then, write the number in decimal form.**

1) $\frac{3}{4} = \frac{\square}{100} = $ ______

2) $\frac{2}{5} = \frac{\square}{10} = $ ______

3) $\frac{9}{50} = \frac{\square}{100} = $ ______

4) $\frac{11}{20} = \frac{\square}{100} = $ ______

5) $\frac{4}{5} = \frac{\square}{10} = $ ______

6) $\frac{12}{25} = \frac{\square}{100} = $ ______

• Converting Fractions to Terminating Decimals • ☆

In each exercise, find the equivalent fraction with a denominator of 10 or 100. Then, write the number in decimal form.

1) $\frac{3}{5}$ = ________ **2)** $\frac{1}{20}$ = ________ **3)** $\frac{2}{25}$ = ________

4) $\frac{13}{20}$ = ________ **5)** $\frac{11}{25}$ = ________ **6)** $\frac{4}{5}$ = ________

7) $\frac{24}{25}$ = ________ **8)** $\frac{47}{50}$ = ________ **9)** $\frac{11}{20}$ = ________

10) $\frac{2}{5}$ = ________ **11)** $\frac{9}{20}$ = ________ **12)** $\frac{22}{25}$ = ________

13) $\frac{27}{50}$ = ________ **14)** $\frac{16}{25}$ = ________ **15)** $\frac{7}{20}$ = ________

• Converting Fractions to Terminating Decimals • ☆

If the denominator of a fraction does not divide evenly into 10 or 100, we divide the denominator *into* the numerator. Your answer will either be a **TERMINATING DECIMAL** or a **REPEATING DECIMAL**. Here, we will work with **TERMINATING DECIMALS**.

EXAMPLE: **Write $\frac{3}{8}$ as a decimal fraction.**

8 does not divide evenly into 10 or 100, so we will have to divide (3 ÷ 8).

Rewrite $\frac{3}{8}$ as a long division problem and solve.

$\frac{3}{8}$ can be read as "3 divided by 8" or $8\overline{)3}$

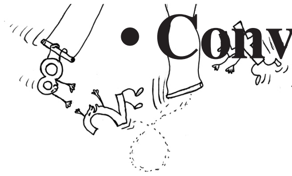
$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\,4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$\frac{3}{8}$ = **0.375**

Try these: **Write as a decimal fraction.**

1) $\frac{5}{8}$ = __________

2) $\frac{3}{16}$ = __________



• Converting Fractions to Terminating Decimals • ☆

Write as a decimal fraction.

1) $\frac{1}{8}$ = ____________

2) $\frac{3}{40}$ = ____________

3) $\frac{5}{16}$ = ____________

4) $\frac{7}{8}$ = ____________

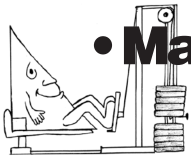
5) $\frac{13}{16}$ = ____________

6) $\frac{3}{8}$ = ____________

7) $\frac{1}{16}$ = ____________

8) $\frac{7}{40}$ = ____________

9) $\frac{11}{32}$ = ____________



• Mastery Check: Converting Fractions to Terminating Decimals •

Write as a decimal fraction.

1) $\frac{3}{4}$ = ____________

2) $\frac{7}{8}$ = ____________

3) $\frac{3}{5}$ = ____________

4) $\frac{13}{40}$ = ____________

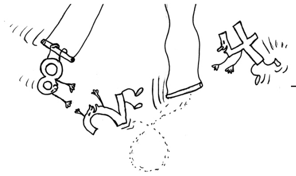
5) $\frac{11}{20}$ = ____________

6) $\frac{5}{16}$ = ____________

Challenge:

7) $\frac{37}{32}$ = ______________

8) $\frac{9}{250}$ = ______________



• Three Types of Decimals •

☆

When you convert a fraction to a decimal by dividing the denominator into the numerator, the answer will be one of the following three things:

1. TERMINATING DECIMAL (The decimal digits end.)

$\frac{1}{4} = 0.25$ $\frac{3}{8} = 0.375$ $\frac{2}{5} = 0.4$

2. PURE REPEATING DECIMAL (All the decimal digits repeat.)

$\frac{1}{3} = 0.\overline{3}$ $\frac{1}{7} = 0.\overline{142857}$ $\frac{7}{11} = 0.\overline{63}$

3. PARTIALLY REPEATING DECIMAL (Some of the decimal digits repeat.)

$\frac{1}{6} = 0.1\overline{6}$ $\frac{1}{12} = 0.08\overline{3}$ $\frac{7}{30} = 0.2\overline{3}$

Try these:

1) Circle the **PURE REPEATING DECIMALS**.

0.51 $0.\overline{7}$ $0.8\overline{56}$ 0.125 $0.1\overline{6}$ $0.\overline{92}$

2) Circle the **TERMINATING DECIMALS**.

$0.\overline{36}$ $0.41\overline{6}$ 0.625 $0.8\overline{3}$ $0.\overline{1}$ 0.97

3) Circle the **PARTIALLY REPEATING DECIMALS**.

0.9 $0.\overline{24}$ $0.41\overline{6}$ $0.\overline{04}$ $0.2\overline{9}$ 0.875

• Three Types of Decimals •

1) Circle the **Pure Repeating Decimals**.

$0.\overline{3}$ $0.7\overline{12}$ 0.625 0.1125 $0.\overline{58}$ 0.1

2) Circle the **Terminating Decimals**.

$0.\overline{81}$ 0.8 0.556 $0.0\overline{1}$ 0.123 $0.\overline{27}$

3) Circle the **Partially Repeating Decimals**.

$0.6\overline{3}$ 0.4 0.875 $0.1\overline{35}$ $0.3\overline{1}$ $0.\overline{384}$

4) Circle the **Terminating Decimals**.

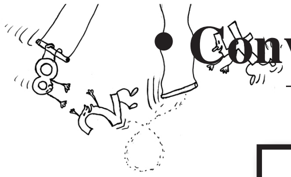
0.5 $0.\overline{18}$ 0.333 $0.\overline{98}$ $0.3\overline{5}$ 0.00125

5) Circle the **Pure Repeating Decimals**.

0.66 $0.\overline{6}$ $0.1\overline{58}$ 0.25 0.5 $0.\overline{142857}$

6) Circle the **Partially Repeating Decimals**.

$0.9\overline{3}$ 0.61 $0.\overline{25}$ $0.1\overline{6}$ 0.4162 $0.\overline{6}$



• Converting Fractions to Pure Repeating Decimals •

Direct Teaching

Converting Fractions to Pure Repeating Decimals

As you divide, if one of the remainders is the **same** as the **DIVIDEND**, you can stop dividing because ***all of the digits*** in the quotient ***repeat***.

Example 1: Convert $\frac{1}{3}$ to a decimal.

$$\begin{array}{r} 0.3 \\ 3\overline{)1.0} \\ -\ \ 9 \\ \hline 1 \end{array}$$

DIVIDEND

REMAINDER

Since the remainder is the same as the dividend, *the decimal digit* in the quotient (3) repeats.

So, $\frac{1}{3}$ as a decimal is $0.\overline{3}$

Example 2: Convert $\frac{7}{11}$ to a decimal.

$$\begin{array}{r} 0.63 \\ 11\overline{)7.00} \\ -6\,6\ \ \\ \hline 40 \\ -33 \\ \hline 7 \end{array}$$

DIVIDEND

REMAINDER

Since the remainder is the same as the dividend, *all of the decimal digits* in the quotient (63) repeat.

So, $\frac{7}{11}$ as a decimal is $0.\overline{63}$

Try these: **Convert each fraction to a decimal.**

1) $\frac{7}{9}$ = ________ **2)** $\frac{5}{11}$ = ________ **3)** $\frac{2}{3}$ = ________

Direct Teaching

Converting Fractions to Partially Repeating Decimals

As you divide, if one of the remainders is the **same** as any previous remainder, you can stop dividing because ***all of the digits*** in the quotient will repeat from the *first time you see the remainder* to the *second time you see the remainder*.

Example 1: Convert $\frac{1}{6}$ to a decimal.

$$\begin{array}{r} 0.16 \\ 6\overline{)1.00} \\ -\;\;6 \\ \hline 40 \\ -36 \\ \hline 4 \end{array}$$

FIRST REMAINDER of 4

SECOND REMAINDER of 4

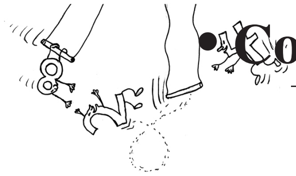
Since the **REMAINDER of 4** repeats, *only* the "6" in the quotient *repeats*.

So, $\frac{1}{6}$ as a decimal is $0.1\overline{6}$

Try these: **Convert each fraction to a decimal.**

1) $\frac{5}{6}$ = __________

2) $\frac{4}{15}$ = __________



• Converting Fractions to Repeating Decimals •

Circle your answer.

1) If one of the remainders is the ***same*** as the ***previous*** remainder:

   a) The last digit in the quotient repeats.
   b) All of the digits in the quotient repeat.
   c) None of the digits in the quotient repeat.

2) If one of the remainders is the ***same*** as the ***dividend***:

   a) The last digit in the quotient repeats.
   b) All of the digits in the quotient repeat.
   c) None of the digits in the quotient repeat.

Convert each fraction to a decimal.

3) $\frac{1}{12}$ = ________

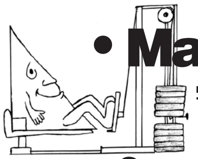
4) $\frac{4}{9}$ = ________

5) $\frac{9}{11}$ = ________

6) $\frac{7}{15}$ = ________

7) $\frac{2}{3}$ = ________

8) $\frac{11}{30}$ = ________



• Mastery Check: Converting Fractions to Repeating Decimals •

Convert each fraction to a decimal.

1) $\frac{3}{11} =$ ________ **2)** $\frac{1}{6} =$ ________ **3)** $\frac{5}{9} =$ ________

4) $\frac{7}{30} =$ ________ **5)** $\frac{5}{6} =$ ________ **6)** $\frac{1}{7} =$ ____________

Circle your answer.

7) If one of the remainders is the ***same*** as the ***previous*** remainder:

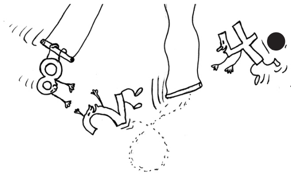
- **a)** The last digit in the quotient repeats.
- **b)** All of the digits in the quotient repeat.

8) If one of the remainders is the ***same*** as the ***dividend***:

- **a)** The last digit in the quotient repeats.
- **b)** All of the digits in the quotient repeat.

Challenge:

9) $\frac{77}{600} =$ ____________



• Converting Decimals to Fractions •

EXAMPLE: Write the decimal fraction 0.14 as a COMMON FRACTION in lowest terms.

STEP 1: Write the number in **SHORT WORD FORM**.

14 hundredths

SHORT WORD FORM

STEP 2: Write the number as a **COMMON FRACTION**.

$$\frac{\boxed{14}}{\boxed{100}}$$

COMMON FRACTION

STEP 3: Reduce to ***lowest terms*** if necessary.

$$\frac{14}{100} = \frac{\mathbf{7}}{\mathbf{50}}$$

Try these: **Write each decimal fraction as a COMMON FRACTION in lowest terms.**

1) 0.75 = | 75 hundredths | (SHORT WORD FORM) = $\frac{\boxed{}}{\boxed{}}$ (COMMON FRACTION)

0.75 as a **COMMON FRACTION** in ***lowest terms*** = ______

---

2) 0.08 = | | (SHORT WORD FORM) = $\frac{\boxed{}}{\boxed{}}$ (COMMON FRACTION)

0.08 as a **COMMON FRACTION** in ***lowest terms*** = ______

• Converting Decimals to Fractions •

Write each decimal fraction as a COMMON FRACTION in lowest terms.

1) 0.45 = ______________ = ―――

SHORT WORD FORM

COMMON FRACTION

0.45 as a **COMMON FRACTION** in ***lowest terms*** = ______

---

2) 0.02 = ______________ = ―――

SHORT WORD FORM

COMMON FRACTION

0.02 as a **COMMON FRACTION** in ***lowest terms*** = ______

Write each decimal fraction as a COMMON FRACTION in lowest terms.

3) 0.3 = ______ **4)** 0.22 = ______ **5)** 0.77 = ______

6) 0.35 = ______ **7)** 0.83 = ______ **8)** 0.12 = ______

9) 0.75 = ______ **10)** 0.28 = ______ **11)** 0.9 = ______

12) 0.26 = ______ **13)** 0.73 = ______ **14)** 0.88 = ______

• Converting Decimals to Fractions •

When converting a decimal that is ***greater than* 1** to a fraction, the value of the fraction is greater than 1 and can be written as a **MIXED NUMBER**.

EXAMPLE: Write the decimal fraction 1.35 as a MIXED NUMBER in lowest terms.

STEP 1: Write the number in **SHORT WORD FORM**.
Note: The decimal point is written as "and."

1 and 35 hundredths

SHORT WORD FORM

STEP 2: Write the number as a **MIXED NUMBER**.

$$1\frac{35}{100}$$

MIXED NUMBER

STEP 3: Reduce to ***lowest terms*** if necessary.

$$1\frac{35}{100} = 1\frac{7}{20}$$

Try these: **Write each decimal fraction as a MIXED NUMBER in lowest terms.**

1) 1.25 = ______________ = ______

SHORT WORD FORM

MIXED NUMBER

1.25 as a **MIXED NUMBER** in ***lowest terms*** = ________

2) 2.8 = ______________ = ______

SHORT WORD FORM

MIXED NUMBER

2.8 as a **MIXED NUMBER** in ***lowest terms*** = ________

• Converting Decimals to Fractions •

Write each decimal as a MIXED NUMBER in lowest terms.

1) 1.4 = ____________ = ____

SHORT WORD FORM — MIXED NUMBER

1.4 as a **MIXED NUMBER** in ***lowest terms*** = ________

---

2) 6.75 = ____________ = ____

SHORT WORD FORM — MIXED NUMBER

6.75 as a **MIXED NUMBER** in ***lowest terms*** = ________

Write each decimal as a MIXED NUMBER in lowest terms.

3) 3.1 = ________ **4)** 4.22 = ________ **5)** 5.05 = ________

6) 9.8 = ________ **7)** 3.14 = ________ **8)** 7.88 = ________

9) 1.42 = ________ **10)** 10.35 = ________ **11)** 6.08 = ________

12) 3.76 = ________ **13)** 2.5 = ________ **14)** 8.26 = ________

• Converting Decimals to Fractions •

Convert each decimal to a COMMON FRACTION or a MIXED NUMBER in lowest terms.

1) 1.65 = ________

2) 0.03 = ________

3) 5.8 = ________

4) 2.77 = ________

5) 12.32 = ________

6) 0.66 = ________

7) 0.02 = ________

8) 2.95 = ________

9) 0.19 = ________

10) 9.51 = ________

11) 4.38 = ________

12) 0.999 = ________

13) 0.5 = ________

14) 8.23 = ________

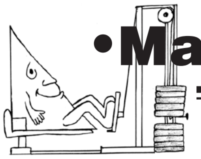
15) 6.82 = ________

16) 8.16 = ________

17) 10.45 = ________

18) 0.024 = ________

Bypass



•Mastery Check: Converting Decimals to Fractions•

Convert each decimal to a COMMON FRACTION or a MIXED NUMBER in lowest terms.

1) 0.07 = ________ **2)** 2.3 = ________ **3)** 5.08 = ________

4) 4.26 = ________ **5)** 0.35 = ________ **6)** 0.03 = ________

7) 0.12 = ________ **8)** 4.7 = ________ **9)** 0.8 = ________

Challenge: **Convert each decimal to a COMMON FRACTION or a MIXED NUMBER in lowest terms.**

10) 0.002 = __________ **11)** 0.0495 = __________

• Converting Percents to Fractions •

Percent means "***for each 100***."

EXAMPLE 1: Convert 7% to a fraction in lowest terms.

7% means, "**7** ***for each 100***," which is the same as $\frac{7}{100}$.

$7\% = \boxed{7}$ for each $100 = \dfrac{\boxed{7}}{100}$

7% as a fraction in lowest terms = $\mathbf{\dfrac{7}{100}}$

EXAMPLE 2: Convert 16% to a fraction in lowest terms.

16% means, "**16** ***for each 100***," which is the same as $\frac{16}{100}$.

$16\% = \boxed{16}$ for each $100 = \dfrac{\boxed{16}}{100}$

16% as a fraction in lowest terms = $\mathbf{\dfrac{4}{25}}$

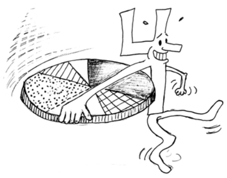
Try these: **Convert each percent to a fraction in lowest terms.**

1) $11\% = \boxed{}$ for each $100 = \dfrac{\boxed{}}{100}$

11% as a fraction in lowest terms = ______

2) $24\% = \boxed{}$ for each $100 = \dfrac{\boxed{}}{100}$

24% as a fraction in lowest terms = ______



• Converting Percents to Fractions •

Convert each percent to a fraction in lowest terms.

1) 32% = $\square$ for each 100 = $\frac{\square}{100}$

32% as a fraction in lowest terms = ______

2) 75% = $\square$ for each 100 = $\frac{\square}{100}$

75% as a fraction in lowest terms = ______

Convert each percent to a fraction in lowest terms.

3) 30% = _____ **4)** 12% = _____ **5)** 8% = _____

6) 25% = _____ **7)** 60% = _____ **8)** 15% = _____

9) 10% = _____ **10)** 2% = _____ **11)** 85% = _____

12) 56% = _____ **13)** 22% = _____ **14)** 67% = _____

15) 18% = _____ **16)** 59% = _____ **17)** 98% = _____

• Converting Percents to Fractions •

☆

When converting a percent that is greater than 100% to a fraction, the value of the fraction is greater than 1 and can be written as a **MIXED NUMBER**.

EXAMPLE: Convert 240% to a MIXED NUMBER in lowest terms.

240% means, "**240 *for each 100***," which is the same as $\frac{240}{100}$.

$240\% = \boxed{240}$ for each $100 = \frac{\boxed{240}}{100}$

240% as a **MIXED NUMBER** in lowest terms = $\underline{2\frac{2}{5}}$

Try these: **Convert each percent to a MIXED NUMBER in lowest terms.**

1) $125\% = \boxed{}$ for each $100 = \frac{\boxed{}}{100}$

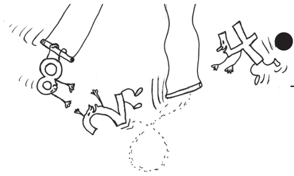
125% as a **MIXED NUMBER** in lowest terms = ______

2) $350\% = \boxed{}$ for each $100 = \frac{\boxed{}}{100}$

350% as a **MIXED NUMBER** in lowest terms = ______

3) $212\% = \boxed{}$ for each $100 = \frac{\boxed{}}{100}$

212% as a **MIXED NUMBER** in lowest terms = ______



• Converting Percents to Fractions •

Convert each percent to a *proper fraction* or a MIXED NUMBER in lowest terms.
Note: Answers can also be WHOLE NUMBERS.

1) 185% = ______ **2)** 322% = ______ **3)** 58% = ______

4) 305% = ______ **5)** 632% = ______ **6)** 72% = ______

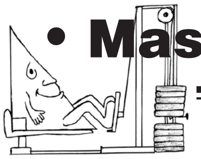
7) 3% = ______ **8)** 210% = ______ **9)** 66% = ______

10) 124% = ______ **11)** 45% = ______ **12)** 40% = ______

13) 55% = ______ **14)** 88% = ______ **15)** 714% = ______

16) 802% = ______ **17)** 1000% = ______ **18)** 34% = ______

19) 318% = ______ **20)** 600% = ______ **21)** 4% = ______



• Mastery Check: Converting Percents to Fractions • ☆

Convert each percent to a *proper fraction* or a MIXED NUMBER in lowest terms.

1) 9% = _____

2) 123% = _____

3) 80% = _____

4) 56% = _____

5) 135% = _____

6) 24% = _____

7) 210% = _____

8) 29% = _____

9) 15% = _____

10) 95% = _____

11) 14% = _____

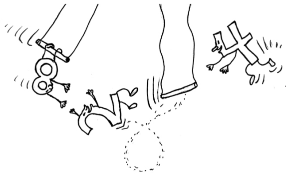
12) 430% = _____

Challenge:

Convert each percent to a *proper fraction* or a MIXED NUMBER in lowest terms.

13) 7.5% = _______

14) 5,005% = _______



• Percent of Change •

When the value of a quantity (the price of an item, a person's weight, the water level in a hole) changes, the ***amount of the change*** is the difference between the original and the new amount.

For example, when the price of a t-shirt drops from \$20 to \$17, the ***amount of change*** is \$3. (\$20 − \$17 = \$3)

The **Percent of Change** is the part of the original amount that the change represents. It is found by dividing the ***amount of change*** by the ***original amount***.

$$\textbf{Percent of Change} = \frac{\text{Amount of Change}}{\text{Original Amount}}$$

Example: The price of a t-shirt dropped from \$20.00 to \$17.00. What is the percent of the discount in price?

$$\textbf{Percent of Change} = \frac{\text{Amount of Change}}{\text{Original Amount}} = \frac{(20 - 17)}{20} = \frac{3}{20}$$

$\frac{3}{20}$ written as a percent is 15%.

So, the percent of the discount in price is **15%**.

Try these:

1) The price of a guitar dropped from \$400 to \$320. Find the percent of the discount in price.

____

2) The price of a gallon of gasoline increased from \$3.00 to \$4.00 over a period of 3 months. Find the percent of increase in cost.

____

• Percent of Change •

☆

$$\text{PERCENT OF CHANGE} = \frac{\text{Amount of Change}}{\text{Original Amount}}$$

1) The price of a wallet went from \$10.00 to \$11.00. Find the percent of increase.

____

2) A coat that usually costs \$50.00 is on sale for \$43.00. Find the percent of the discount.

____

3) The price of a meal went from \$16.00 to \$18.00. Find the percent of increase.

____

4) A bicycle tire that usually costs \$18.00 is on sale for \$10.00. Find the percent of the discount.

____

5) The price of a toy went from \$12.00 to \$15.00. Find the percent of increase.

____

• Percent of Change •

$$\text{PERCENT OF CHANGE} = \frac{\text{Amount of Change}}{\text{Original Amount}}$$

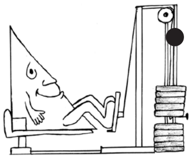
1) A car that usually costs $50,000 is on sale for $49,000. Find the percent of the discount.

2) The price of a ticket went from $15.00 to $24.00. Find the percent of increase.

3) A computer that usually costs $1,500 is on sale for $1,250. Find the percent of the discount.

4) The price of a chair went from $16.00 to $20.00. Find the percent of increase.

5) The number of students in a school increased from 300 to 324 last year. Find the percent of increase.



• Mastery Check: Percent of Change •

1) The price of a CD increased from $20 to $24. Find the percent of increase.

____

2) A shirt that usually costs $30 is on sale for $20. Find the percent of the discount.

____

3) A puppy weighed 25 ounces. One week later, it weighed 36 ounces. Find the percent of increase.

____

4) A baseball glove that usually costs $90 is on sale for $70. Find the percent of the discount.

____

Challenge:

5) After a snow storm, the snow was 2 feet deep. Three days later, the snow was only 14 inches deep. Find the percent of decrease in snow depth.

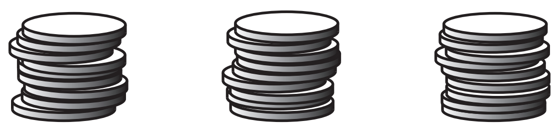
____

• Evening Out the Stacks •

Example:

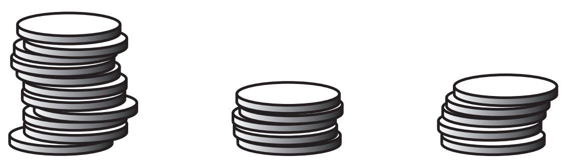
There are **12** coins in the first stack, **5** in the second stack, and **10** in the third stack. If the coins are rearranged so that each of the three stacks has the same number of coins, how many coins will be in each stack?

First, combine all three stacks into one pile. Since **12 + 5 + 10 = 27**, we have **27** coins in this pile. Now, let's redistribute them evenly back into the three stacks. Distribute one coin to each stack. Then distribute another coin to each stack. Keep going until we run out of coins. We now have **9** coins in each stack.



***Try these*:**

1)



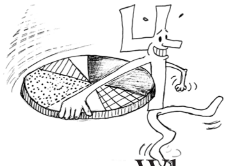
There are 11 coins in the first stack, 5 in the second stack, and 5 in the third stack. If the coins are rearranged so that each of the three stacks has the same number of coins, how many coins will be in each stack?

____

2)

There are 3 coins in the first stack, 8 in the second stack, and 1 in the third stack. If the coins are rearranged so that each of the three stacks has the same number of coins, how many coins will be in each stack?

____



• Finding the Mean •

When we "even out the stacks," we are finding a single number to represent the number of coins in each of our stacks. This number is called the average or the **MEAN**. To find the **MEAN** of a data set, add up all of the values and then divide that sum by the number of values in the data set. This process "evens out" the values. In subjects such as statistics and sociology, finding the **MEAN** of a data set helps us to understand and simplify information.

Direct Teaching

EXAMPLE:

Find the mean of the following data set:

$$\{\mathbf{12},\ \mathbf{5},\ \mathbf{10}\}$$

Begin by finding the sum of the values (combining them into one pile).

$$\mathbf{12 + 5 + 10 = 27}$$

Now, take this sum and divide it by **3** (redistribute the pile into **3** stacks), the number of values we have.

$$\text{MEAN} = \frac{\text{sum of values}}{\text{\# of values}} = \frac{\mathbf{27}}{\mathbf{3}} = \mathbf{9}$$

So, the **MEAN** of the data set is **9** (there are **9** in each of the **3** stacks).

***Try these*: Find the mean of the following data sets.**

1) {3, 2, 1, 6, 8}

What is the sum of the values in the data set? ☐

How many values are there in the data set? ☐

$$\text{MEAN} = \frac{\text{sum of values}}{\text{\# of values}} = \frac{\square}{\square} = \underline{\hspace{3em}}$$

2) {9, 20, 1, 17, 14, 5, 25}

What is the sum of the values in the data set? ☐

How many values are there in the data set? ☐

$$\text{MEAN} = \frac{\text{sum of values}}{\text{\# of values}} = \frac{\square}{\square} = \underline{\hspace{3em}}$$

• Finding the Mean •

☆

Find the mean of the following data sets.

1) {10, 12, 4, 6, 18, 2, 4}

What is the sum of the values in the data set? $\square$

How many values are there in the data set? $\square$

$$\textbf{MEAN} = \frac{\text{sum of values}}{\text{\# of values}} = \frac{\square}{\square} = \underline{\hspace{3em}}$$

2) {5, 16, 4, 8, 5, 4}

What is the sum of the values in the data set? $\square$

How many values are there in the data set? $\square$

$$\textbf{MEAN} = \frac{\text{sum of values}}{\text{\# of values}} = \frac{\square}{\square} = \underline{\hspace{3em}}$$

---

3) {0, 17, 8, 7, 3}

MEAN = __________

4) {5, 21, 19, 4, 1}

MEAN = __________

5) {1, 1, 10}

MEAN = __________

• Finding the Mean •

☆

1) A seventh grade class got their math tests back. The teacher wanted the whole class to have an average score of at least 80. The individual scores were:

{99, 50, 92, 94, 55}.

Did the class reach the teacher's goal? Find the mean and explain.

____

____

2) Gioia owns a pizzeria and wants to find out the average number of slices of pizza per person she sells during a typical lunch hour. A total of 14 people came in during lunch today. There were four orders of 5 slices, five orders of 3 slices, two orders of 2 slices, and three orders of 1 slice. What is the average number of slices of pizza per person that Gioia's pizzeria sold during lunch?

____

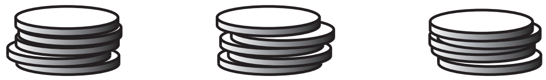
3) Pooja and Platini both took the same four math tests. Pooja scored the following on the tests: {98, 82, 92, 100}. Platini scored {95, 98, 68, 99}. Who had the better test score average? Explain.

____

____

• Finding a Value •

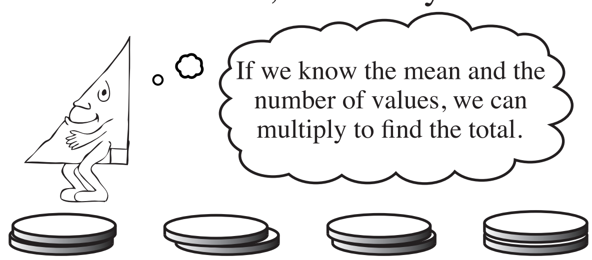
1) **a)** If there are 3 stacks of 5 coins, how many coins are there in total?



5 **+** **5** **+** **5**

____

b) If there are 4 stacks of 2 coins, how many coins are there in total?



____

2) **a)** If the scores of 3 tests are 63, 95, and 92, how many points have been scored so far?

____

b) If the scores of 4 tests are 96, 74, 80, and 76, how many points have been scored so far?

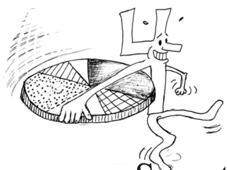
____

3) **a)** Earl has his last Algebra test coming up. If he currently has 652 points in the class and needs 726 points to pass the class, what must Earl score on his final test?

____

b) Susan has her last Geometry test coming up. If she currently has 345 points in the class and needs 428 points to pass the class, what must Susan score on her final test?

____



• Finding a Value •

☆

Wholes and Parts

Sometimes we need to find a part of the data set, given the other parts and the MEAN.

EXAMPLE: Kevin's math grade will be the average of five tests given throughout the semester. He has already taken four of them. His scores are:

{**75**, **90**, **88**, **97**, ____}.

What score does Kevin need to get on his last test in order to have an average of **90**?

Step 1:

First, we need to find the total number of points Kevin needs in order to have an average of **90** on **5** tests. Since we know that the mean is **90** and that there are **5** tests, we can find the total by multiplying.

Total points required:

$\underbrace{\mathbf{90}}_{\text{mean}} \times \mathbf{5} = 450$

Step 2:

Next, find the total number of points Kevin has so far.

$\mathbf{75 + 90 + 88 + 97 = 350}$

Step 3:

Finally, find how many more points Kevin needs in order to have a total of **450** points. Since the whole (**450**) is the sum of its parts (**350** plus the score on the fifth test), we can now determine what Kevin needs to score on his last math test.

$\mathbf{350} + \underline{100} = 450$

So, Kevin needs to score a **100** on his last test to have an average of **90** in his class.

***Try this*:**

Direct Teaching

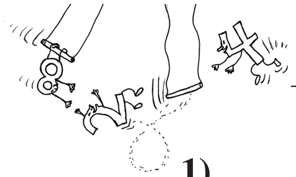
{1, 5, 6, 2, ____}

In the set above, what is the missing value so that the mean is 3?

We need $\underbrace{3}_{\text{mean}} \times \underbrace{5}_{\text{\# of values}} = \square$ in total.

We have $\square$ so far.

$\square + \underbrace{\_\_\_\_}_{\text{missing value}} = \square$ So, the fifth value must be ____.



• Finding a Value • ☆

1) $\{9,\ 1,\ 0,\ 10,\ 5,\ 3,\ 2,\ 0,\ 8,\ \_\_\_\_\}$

In the set above, what is the missing value so that the mean is 5?

We need $\underset{\text{mean}}{\_\_\_\_} \times \underset{\text{\# of values}}{\_\_\_\_} = \boxed{}$ in total.

We have $\boxed{}$ so far.

$\boxed{} + \underset{\text{missing value}}{\_\_\_\_} = \boxed{}$ So, the tenth value must be ____.

2) $\{1,\ 1,\ 2,\ 2,\ 4,\ 2,\ 8,\ \_\_\_\_\}$

In the set above, what is the missing value so that the mean is 6?

8TH VALUE = ________

3) $\{5,\ 6\frac{1}{2},\ \frac{1}{2},\ 1,\ 4,\ 2,\ \_\_\_\_\}$

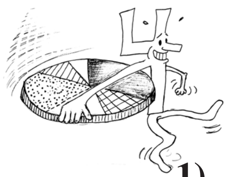
In the set above, what is the missing value so that the mean is 4?

7TH VALUE = ________

---

4) Six friends are having a potluck. For dessert, they all agree to bring cookies so that each one of them can have 3. Two of the friends bring 4 cookies each and three of them bring 2 cookies each. How many cookies does the sixth friend need to bring?

____



• Mean *Practice* •

1) In order to stay in business, a toy store that is usually open everyday must sell 140 toys each week.

a) On average, how many toys does the store need to sell per day?

____

b) At the end of the sixth day, the store has sold the following number of toys:

$$\{32,\ 13,\ 25,\ 28,\ 14,\ 15\}$$

How many toys must the store sell on the seventh day?

____

2) The store's actual average is 24 toys sold per day.

a) How many toys does the store sell in a week?

____

b) This week, the store is closed on Monday for Memorial Day. On average, how many toys do they need to sell each day this week to keep up with their usual weekly sales?

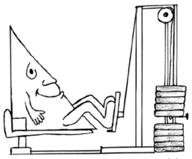
____

c) During the Memorial Day week, the toy store sold the following number of toys:

$$\{14,\ 22,\ 16,\ 21,\ 19,\ 22\}$$

What was the store's average number of toys sold per day?

____



• Mastery Check: Mean •

1) Find the mean of the following data set:

$$\{5,\ 4,\ 20,\ 1,\ 4,\ 6,\ 23\}$$

MEAN = __________

---

2) $\{10,\ 5,\ 4,\ 15,\ 16,\ 11,\ 0,\ 20,\ 19,\ \_\_\_\_\}$

In the set above, what is the missing value so that the mean is 12?

10TH VALUE = __________

---

3) Mark scored an 88 on his first test, an 84 on his second test, a 94 on his third test, and an 86 on his fourth test.

a) What is Mark's average test score for all four tests?

____

b) If Mark wants a test score average of 90, what must he score on his fifth exam?

____

Challenge:

4) Find the mean of the following data set:

$$\{^-2,\ 5,\ 7,\ 9,\ ^-10,\ ^-7,\ ^-9\}$$

MEAN = __________

• Negative Exponents •

Direct Teaching

Any number (except 0) raised to a negative power is the reciprocal of the number to the positive power. For example, 5^{-1} is the same as $\frac{1}{5}$. This is also true for non-zero variables.

Examples: Simplify by rewriting with non-negative exponents.

1) $7^{-3} = \frac{1}{7^3}$

2) $x^{-8} = \frac{1}{x^8}$

3) $\frac{1}{7^{-4}} = \frac{1}{\frac{1}{7^4}} = 1 \div \frac{1}{7^4} = 1 \bullet 7^4 = 7^4$

4) $\frac{1}{x^{-5}} = \frac{1}{\frac{1}{x^5}} = 1 \div \frac{1}{x^5} = 1 \bullet x^5 = x^5$

An expression is considered simplified when there are non-negative exponents.

Try these: **Simplify by rewriting with non-negative exponents.**

1) $6^{-5} =$ ______

2) $y^{-2} =$ ______

3) $h^{-10} =$ ______

4) $\frac{1}{x^{-9}} =$ ______

• Simplifying Expressions with Negative Exponents •

Direct Teaching

When there are multiple negative exponents, we need to take the reciprocal of each one so that all exponents are positive.

EXAMPLE: Simplify $\frac{7a^{-4}}{5b^{-1}}$.

This expression has two negative exponents, so let's take the reciprocal of those variables.

$$\frac{7 \bullet a^{-4}}{5 \bullet b^{-1}} = \frac{7b}{5a^4}$$

So, the expression simplified is $\frac{7b}{5a^4}$.

Recall that an expression is considered simplified when all exponents are non-negative.

***Try these*: Simplify.**

1) $\frac{7b^{-5}}{13c^{-2}} =$ __________

2) $\frac{9x^{-1}}{14y^{-3}} =$ __________

3) $\frac{15}{8h^{-4}k^{-5}} =$ __________

4) $\frac{13q^{-2}s^{-3}}{19r^{-4}t^{-5}} =$ __________

• Simplifying Fractions with Exponents •

Direct Teaching

The general rule for dividing numbers with the same base is:

$$\frac{x^a}{x^b} = x^{a-b}$$

We can use this technique to simplify the exponents.

Example: Simplify $\dfrac{15x^{-5}y^2}{25x^3y^{-4}}$.

$$\frac{15}{25} \text{ reduces to } \frac{3}{5}.$$

$$x^{-5-3} = x^{-8}$$

$$y^{2-(-4)} = y^{2+4} = y^6$$

Our expression is now $\dfrac{3x^{-8}y^6}{5}$. Recall that an expression is considered simplified when all exponents are non-negative. This means that it simplifies to $\dfrac{3y^6}{5x^8}$.

So, $\dfrac{15x^{-5}y^2}{25x^3y^{-4}}$ reduces to $\dfrac{3y^6}{5x^8}$.

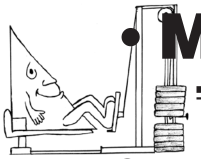
Try these: **Simplify.**

1) $\dfrac{2a^3}{10a^{-7}} =$ __________

2) $\dfrac{12b^{-2}c^2}{16b^6c^{-4}} =$ __________

3) $\dfrac{32y^{-5}z}{18y^2z^{-7}} =$ __________

4) $\dfrac{15t^5u^{-3}}{9u^{-7}v^{-8}} =$ __________



• Mastery Check: Simplifying Fractions •

Simplify if possible.

1) $\dfrac{6a^{-5}b^{4}d^{-1}}{8a^{-3}b^{-2}c^{-2}}$

2) $\dfrac{22m^{-2}n^{-3}}{33g^{-7}h^{-8}}$

3) $\dfrac{7x^{6}y^{-2}}{15x^{-6}}$

4) $\dfrac{15a^{-2}b^{-6}}{96b^{3}c^{5}}$

5) $\dfrac{pq^{-5}}{p^{-1}q^{-3}r}$

6) $\dfrac{14u^{-10}v^{-3}w^{4}}{16u^{10}v^{-3}w^{-1}}$

Challenge:

7) $\dfrac{-25ab^{-1}c^{-3}d^{3}}{10b^{-2}c^{-1}d^{-2}f^{-1}}$

8) $\dfrac{18x^{-4}y^{2}z^{-3}}{(3xy^{4}z^{3})^{-2}}$

• Evaluating Algebraic Expressions •

In algebra, letters are called ***variables*** and they are used to represent numbers. When numbers, variables, and mathematical operations (+, –, ×, ÷) are used together, ***algebraic expressions*** are formed.

EXAMPLES: $\mathbf{2x}$ $\mathbf{3b + 4c}$ $\mathbf{7(a^2 - b)}$ $\mathbf{3x^2 + 5y(z - 8)}$

We often need to ***evaluate*** algebraic expressions when variables are given numerical values.

EXAMPLE: If $\boldsymbol{a} = \mathbf{4}$ and $\boldsymbol{b} = \mathbf{{}^-3}$, evaluate $\boldsymbol{a^2} + \mathbf{6}\boldsymbol{b}$.

Step 1: Replace the variables with the given numerical values.

$$a^2 + 6b$$
$$(\mathbf{4})^2 + 6(\mathbf{{}^-3})$$

Step 2: Perform the indicated operations.

$$(4)^2 + 6({}^-3)$$
$$\mathbf{16} + (\mathbf{{}^-18}) = \mathbf{{}^-2}$$

Try these:

1) If $\boldsymbol{a} = 5$ and $\boldsymbol{b} = 2$, evaluate $\boldsymbol{a} + 3\boldsymbol{b}$. ______________________________

2) If $\boldsymbol{x} = 3$ and $\boldsymbol{y} = 5$, evaluate $\boldsymbol{x}^2 - 2\boldsymbol{y}$. ______________________________

3) If $\boldsymbol{a} = 4$ and $\boldsymbol{b} = 0$, evaluate $\boldsymbol{a} + 3(5\boldsymbol{b}^2 - \boldsymbol{a})$. ______________________________

• Evaluating Algebraic Expressions •

☆

1) If $a = 3$ and $b = 4$, evaluate $2ab + a^2$. ____________________

2) If $x = 5$ and $y = 4$, evaluate $x^2 - xy$. ____________________

3) If $s = {}^{-}2$ and $t = 5$, evaluate $2t^2 + 5s$. ____________________

4) If $a = 6$, $b = 10$, and $c = 3$, evaluate $b^2 - a^2 + c$. ____________________

5) If $m = 10$ and $n = 3$, evaluate $6m + 3mn$. ____________________

---

6) Evaluate $3a - 2b$ when $a = 7$ and $b = 9$. ____________________

7) Evaluate $2a^2 - 3b^2$ when $a = 4$ and $b = 3$. ____________________

8) Evaluate $3(g + h^2)$ when $g = {}^{-}5$ and $h = 3$. ____________________

9) Evaluate $x^2 + 3y - z$ when $x = 6$, $y = {}^{-}7$, and $z = 9$. ____________________

10) Evaluate $z^3 - 4y^2$ when $z = 3$ and $y = 2$. ____________________

• Evaluating Algebraic Expressions •

1) If $s = 3$ and $t = 4$, evaluate $4st - s^2$. ____________________

2) If $x = 3$ and $y = {}^{-}4$, evaluate $x^2 - y$. ____________________

3) If $p = 9$ and $q = 15$, evaluate $3p + 10q$. ____________________

4) If $a = 8$ and $b = 4$, evaluate $3a + b^2$. ____________________

5) If $u = 12$ and $v = 11$, evaluate $3u + 6v - 9$. ____________________

---

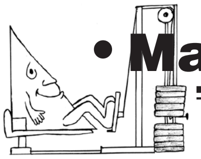
6) Evaluate $5x + 4y - z$ when $x = 4$, $y = {}^{-}2$ and $z = 3$. ____________________

7) Evaluate $12a^2 - 3b$ when $a = 2$ and $b = 15$. ____________________

8) Evaluate $6a + b^3 + 3c$ when $a = {}^{-}3$, $b = 4$, and $c = 2$. ____________________

9) Evaluate $x - y - z$ when $x = {}^{-}1$, $y = {}^{-}2$ and $z = {}^{-}3$. ____________________

10) Evaluate $8z^2 - 8y^2$ when $z = 4$ and $y = 3$. ____________________



• Mastery Check: Evaluating Algebraic Expressions •

1) If $a = 3$ and $b = 4$, evaluate $4a - 5b$. ____________________

2) If $a = 5$, $b = 2$, and $c = 7$, evaluate $3a^2 + 5b(c - 4)$. ____________________

3) If $x = {}^{-}1$ and $y = 4$, evaluate $3x + 2xy$. ____________________

4) Evaluate $a^2 + 2b + ab$ when $a = 5$ and $b = {}^{-}1$. ____________________

5) Evaluate $(a - 3b)^2$ when $a = 10$ and $b = 4$. ____________________

6) Evaluate $(xy + 3) + 8x$ when $x = 4$ and $y = {}^{-}2$. ____________________

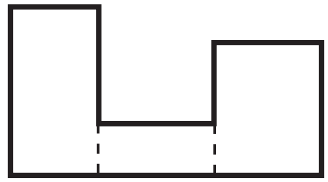
Challenge:

7) If $x = 10$ and $y = {}^{-}5$, evaluate $\dfrac{x - y}{y - x} - \dfrac{x^2}{xy}$. ____________________

• Composite Figures •

Direct Teaching

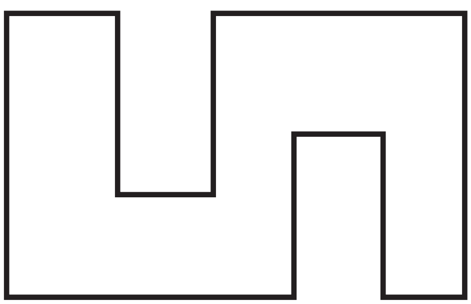
A COMPOSITE FIGURE is a figure that is made up of simple shapes such as rectangles, squares, circles, and triangles.



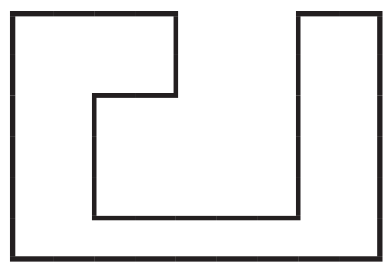
A COMPOSITE FIGURE made up of three rectangles.

Try these: **Split each COMPOSITE FIGURE into simple shapes.**

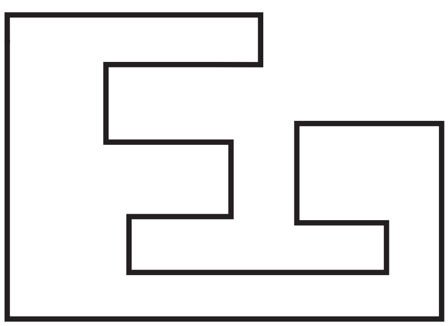
1)



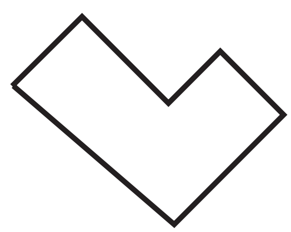
2)



3)



4)

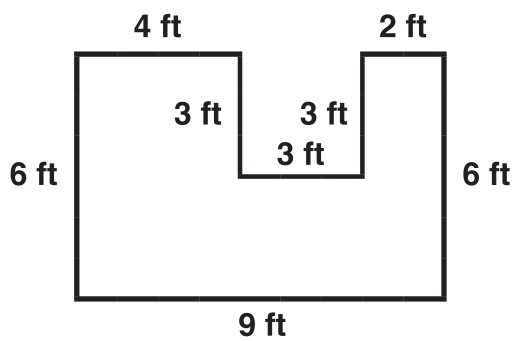


• Perimeter of Composite Figures •

Direct Teaching

To find the perimeter of a composite figure, we add up the lengths of all the sides.

Example: Find the perimeter of the following figure.

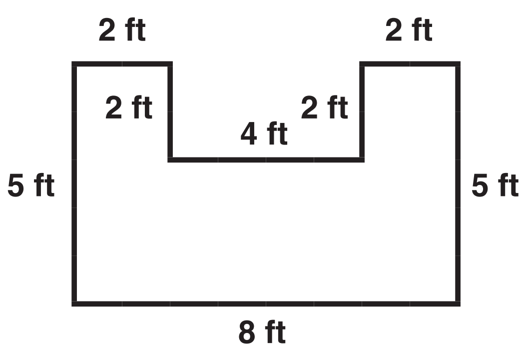


We find the perimeter of the figure by adding the lengths of all the sides together.

So, the perimeter of the figure is $4 + 3 + 3 + 3 + 2 + 6 + 9 + 6 = 36$ feet.

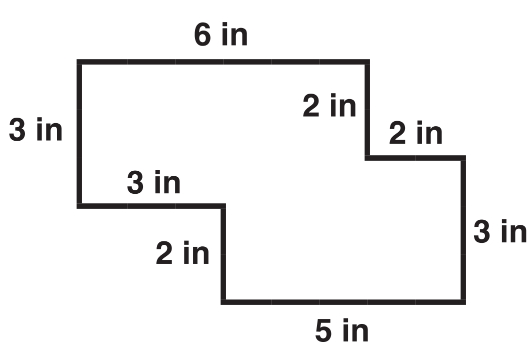
Try these: **Find the perimeters of the following figures.**

1)



PERIMETER = ______________

2)



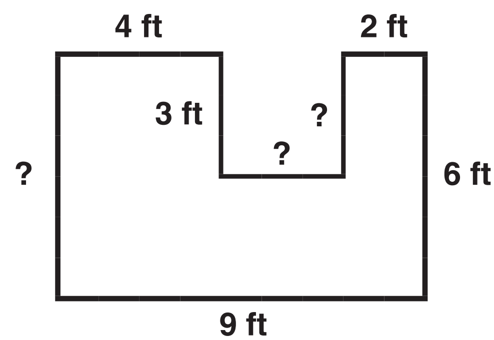
PERIMETER = ______________

• Missing Dimensions of Composite Figures •

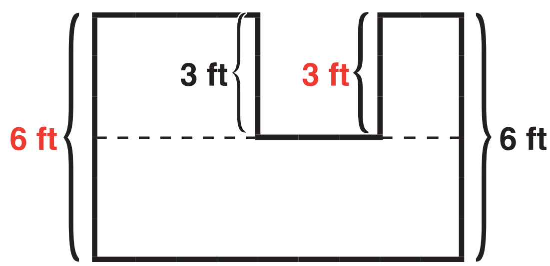
Direct Teaching

Sometimes we are given a composite figure with some missing dimensions. We can use what we are given to figure out the missing dimensions.

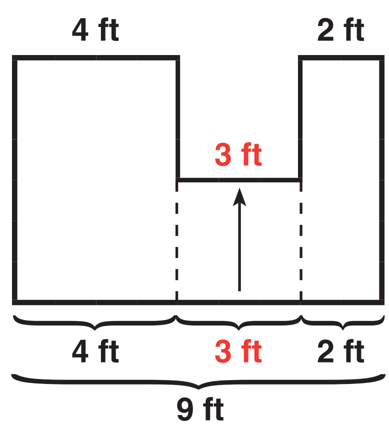
Example: Find the missing dimensions of the following figure.



We can imagine the figure as three separate rectangles to help us find its dimensions.



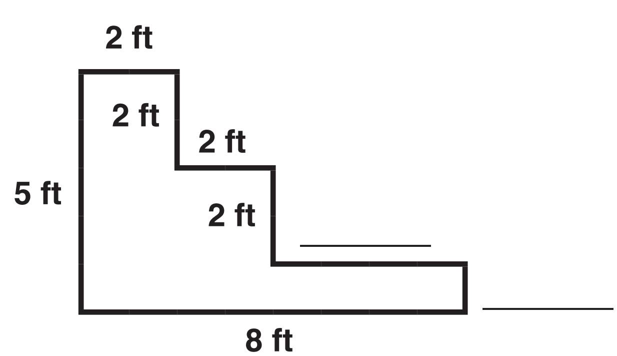
We can find some of the missing dimensions by inspection.



Since we know the bottom of the figure measures 9 feet, we can figure out that the length of the remaining missing dimension is 3 feet.

Try this: **Find the missing dimensions of the following figure.**

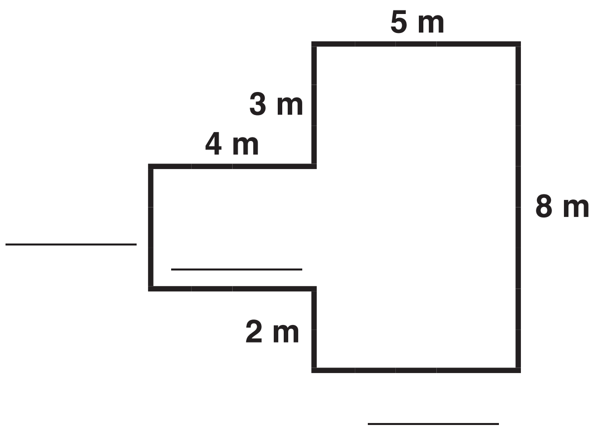
1)



• Missing Dimensions of Composite Figures • ☆

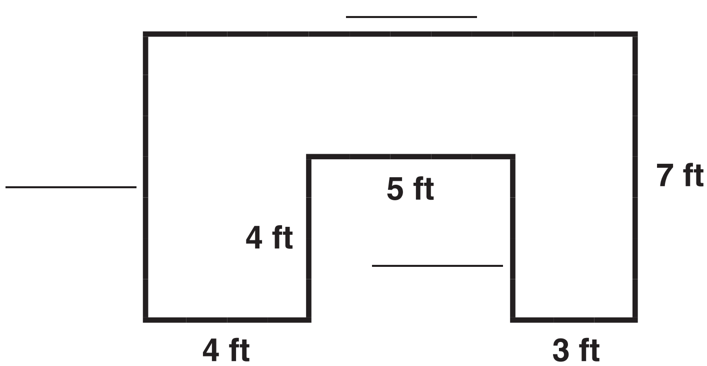
Find the missing dimensions and the perimeters of each figure.

1)



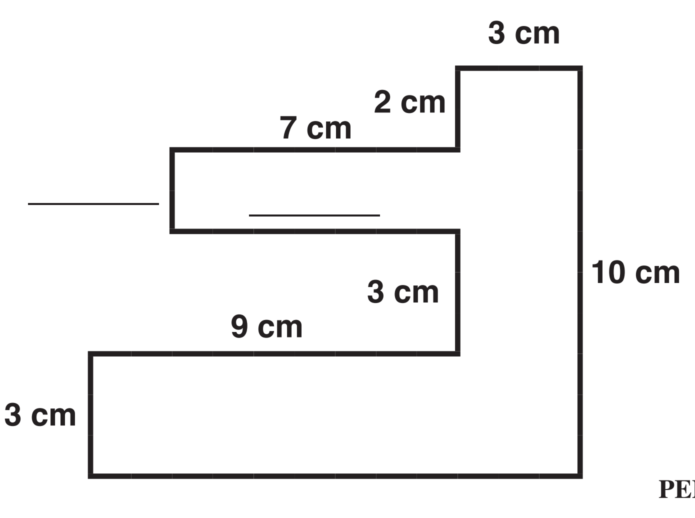
PERIMETER = ________________

2)



PERIMETER = ________________

3)



PERIMETER = ________________

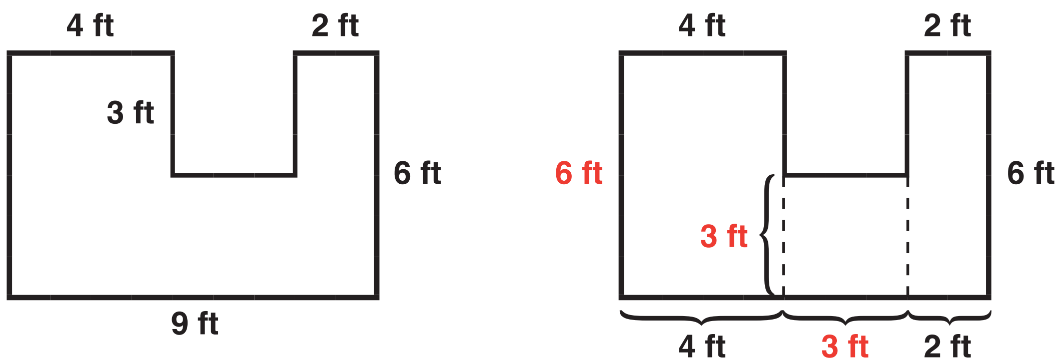
• Area of Composite Figures •

Direct Teaching

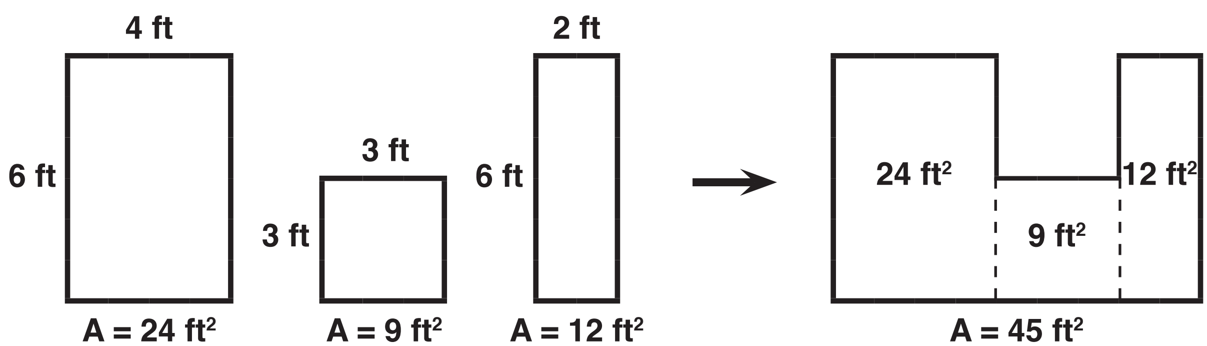
Wholes and Parts

To find the area of a composite figure, we can first split it into simple shapes and find the area of each part.

Example: Find the area of the following figure.



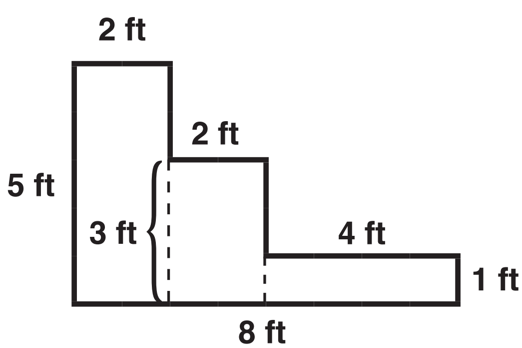
We can imagine this composite figure broken up as three separate rectangles. Because a whole is equal to the sum of its parts, the area of the composite figure is equal to the sum of the areas of the rectangles.



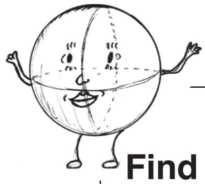
So, the area of the whole figure is $24 + 9 + 12 = 45$ ft^2.

Try this: **Find the area of the following figure.**

1)



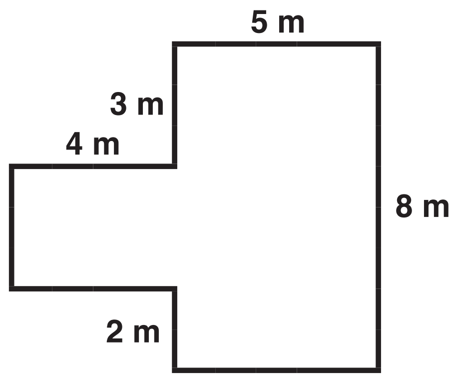
AREA = ______________



• Area of Composite Figures •

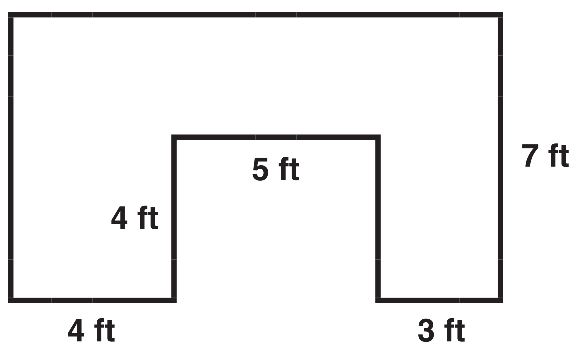
Find the area of each figure.

1)



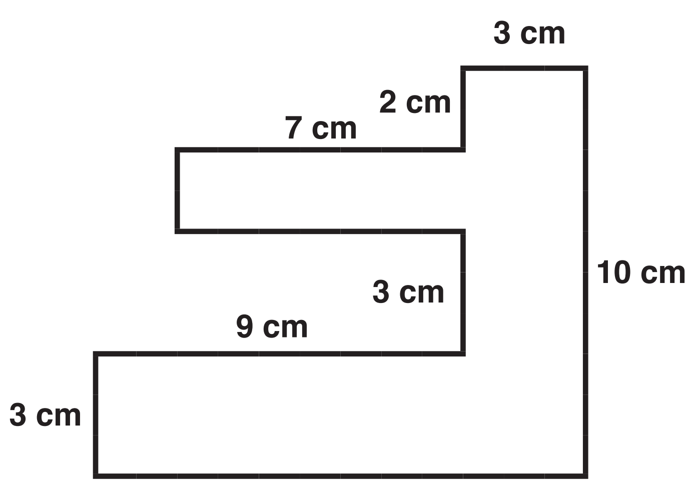
AREA = ________________

2)



AREA = ________________

3)

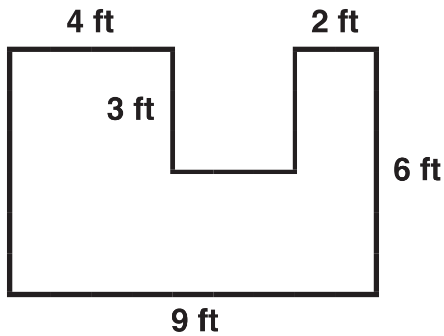


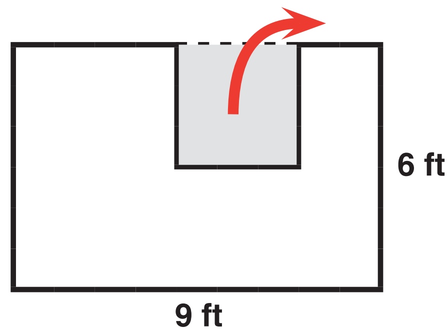
AREA = ________________

• Area of Composite Figures •

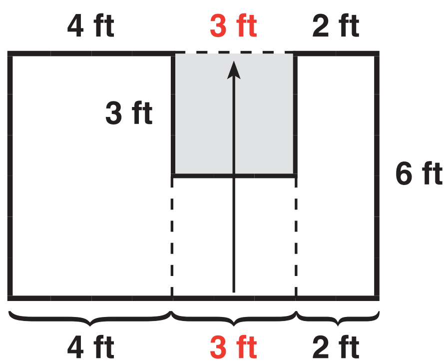
To find the area of a composite figure, we can also subtract a part from a larger whole.

Example: Find the area of the following figure.





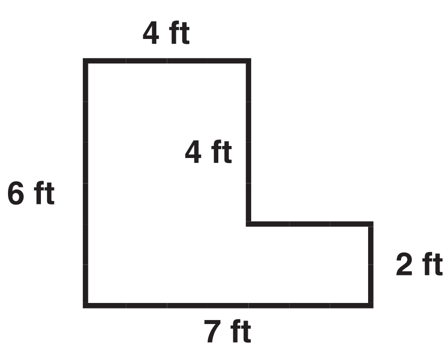
We can imagine this composite figure as a larger rectangle with a square missing from it. In this case, the composite figure is a part of the whole rectangle. So, to find the area of the figure, we subtract the area of the square (9 ft^2) from the area of the rectangle (54 ft^2).



So, the area of the composite figure is 54 ft^2 – 9 ft^2 = 45 ft^2.

Try this: **Find the area of the following figure.**

1)

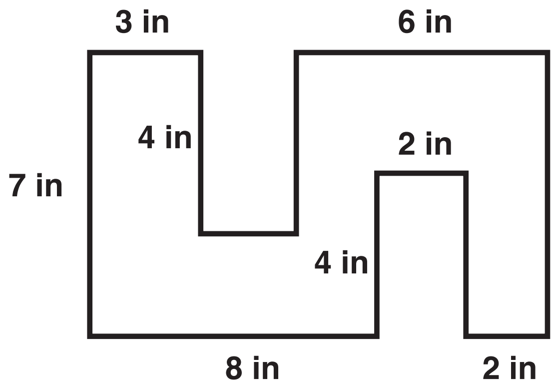


AREA = ____________

• Perimeter and Area of Composite Figures •

Find the perimeter and area of each figure.

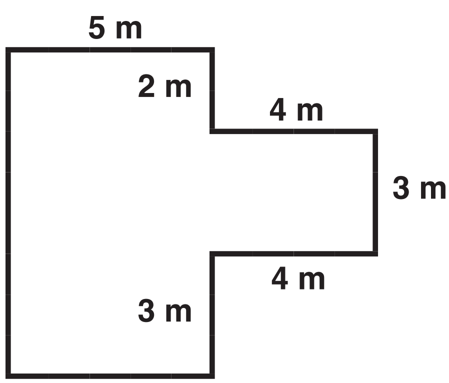
1)



PERIMETER = ______________

AREA = ______________

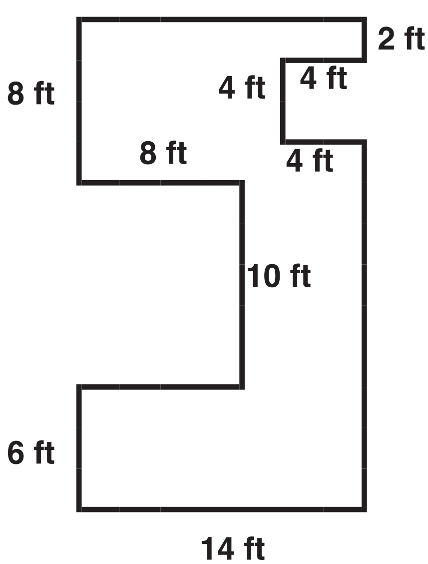
2)



PERIMETER = ______________

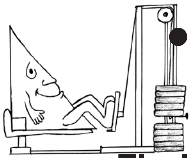
AREA = ______________

3)



PERIMETER = ______________

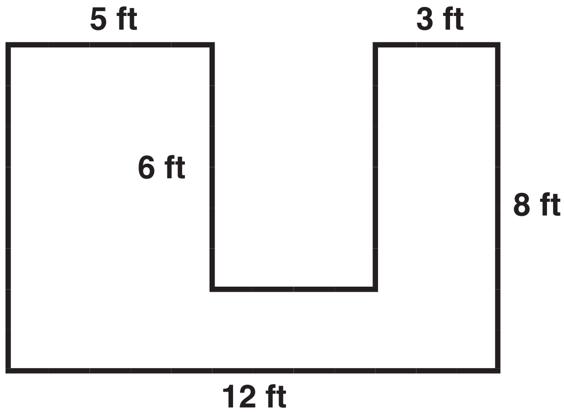
AREA = ______________



• Mastery Check: Composite Figures • ☆

Find the perimeter and area of each figure.

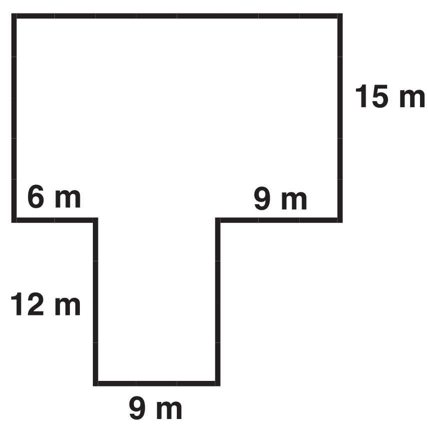
1)



PERIMETER = ________________

AREA = ________________

2)

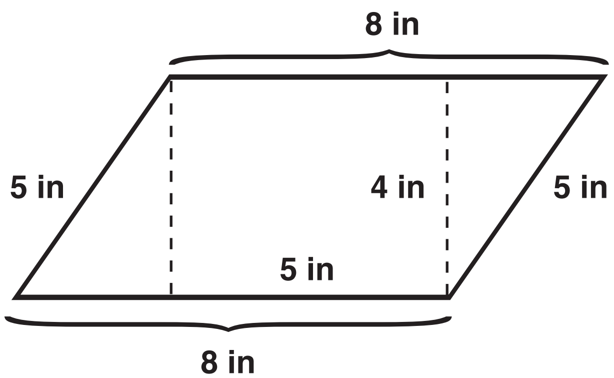


PERIMETER = ________________

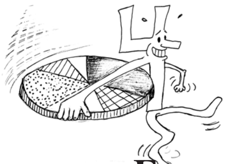
AREA = ________________

Challenge: Find the area.

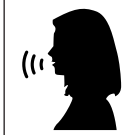
3)



AREA = ________________



• Sums of Factors •


Direct Teaching

Example: What two integers *multiply* to make **12** and *add* to make **$^-8$**?

We can do this by making a chart of factor pairs for **12** and their sums.

Factors	Sum	
1 & 12	13	✗
2 & 6	8	✗
3 & 4	7	✗
$^-1$ & $^-12$	$^-13$	✗
$^-2$ & $^-6$	**$^-8$**	✓
$^-3$ & $^-4$	$^-7$	✗

Since we are looking for factors that *add* to make **$^-8$**, the two integers we are looking for are **$^-2$** and **$^-6$**.

Try these:

1) In the **example** above, is it necessary to list factor pairs that contain positive numbers? YES / NO (circle one)

Explain: ____

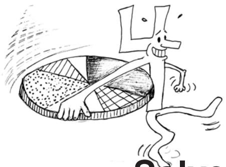
2) Once you get the solution, do you need to keep going on with the list?
YES / NO (circle one)

Explain: ____

3) What two integers *multiply* to make **$^-12$** and *add* to make **4**?

________ & ________

Factors	Sum
1 & $^-12$	



• Sums of Factors •

Solve the following problems. Make a chart if needed.

1) What two integers *multiply* to make **8** and *add* to make **$^{-}6$**?

________ & ________

2) What two integers *multiply* to make **20** and *add* to make **9**?

________ & ________

3) What two integers *multiply* to make **$^{-}12$** and *add* to make **1**?

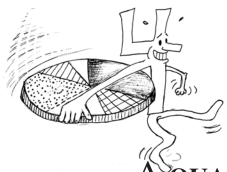
________ & ________

4) What two integers *multiply* to make **$^{-}4$** and *add* to make **0**?

________ & ________

5) What two integers *multiply* to make **17** and *add* to make **18**?

________ & ________



• “Splitting” the Middle Term •

Direct Teaching

A QUADRATIC POLYNOMIAL is a polynomial in the form

$$ax^2 + bx + c$$

where $a \neq 0$ and $\boldsymbol{x}$ is a variable. We factor QUADRATIC POLYNOMIALS because it is a useful method when solving quadratic equations. Finding the two numbers that *multiply* to make $a \bullet c$ and *add* to make b allows us to “split” the middle term and factor the resulting polynomial by grouping.

EXAMPLE: Factor $x^2 + x - 6$.

Steps to Solve:

STEP 1: First, identify the values of a, b, and c.

$a = 1,\ b = 1,\ c = {}^{-}6$

STEP 2: Create a list of all factor pairs for $a \bullet c$ ($1 \bullet {}^{-}6 = \mathbf{{}^{-}6}$) and determine which pair *add* to make b (**1**).

FACTORS OF $a \bullet c$	SUM OF b	
1 & ${}^{-}6$	${}^{-}5$	✗
${}^{-}1$ & 6	5	✗
2 & ${}^{-}3$	${}^{-}1$	✗
${}^{-}2$ & 3	**1**	✓

STEP 3: “Split” the middle term into two terms whose coefficients are the numbers found in **STEP 2**.

NOTE: The order of the middle terms does not matter.

$$x^2 + 1x - 6$$
$$= x^2 + {}^{-}2x + 3x - 6$$

STEP 4: Finally, factor the resulting polynomial by grouping.

$$x^2 - 2x + 3x - 6$$
$$= x(x - 2) + 3(x - 2)$$
$$= (x - 2)(x + 3)$$

So, $x^2 + x - 6$ factored is $(x - 2)(x + 3)$.

Try this: **Identify *a*, *b*, and *c*. Then “split” the middle term and factor by grouping.**

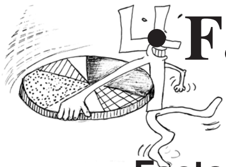
1) $x^2 - 6x + 5$

$a =$ _____, $b =$ _____, $c =$ _____

$x^2 +$ _____ $+$ _____ $+ 5$

$=$ ____________________

FACTORS OF $a \bullet c$	SUM OF b



• Factoring Quadratic Polynomials *Practice* • ☆

Factor the following polynomials.

1) $z^2 - 3z + 2$

= ______________________

2) $x^2 - 24x + 23$

= ______________________

3) $x^2 - 10x + 25$

= ______________________

4) $x^2 + 10x + 21$

= ______________________

---

5) If $a \cdot c$ is positive, the signs of the factors must be THE SAME / DIFFERENT (circle one).

Explain: ____

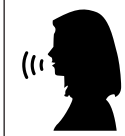
____

6) If $a \cdot c$ is negative, the signs of the factors must be THE SAME / DIFFERENT (circle one).

Explain: ____

____

• Factoring Quadratic Polynomials •



When factoring a QUADRATIC POLYNOMIAL where $a = 1$, the factored polynomial is always in the form

$$(x + \_\_\_\_)(x + \_\_\_\_)$$

where the blank spaces are filled in with the numbers that *multiply* to make $a \bullet c$ and *add* to make b. Let's factor a QUADRATIC POLYNOMIAL *without* "splitting" the middle term and grouping.

EXAMPLE: Factor $x^2 - 5x + 4$.

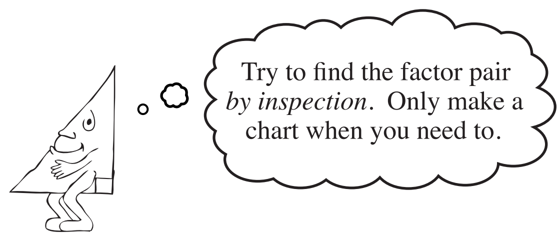
Steps to Solve:

STEP 1: First, identify the values of a, b, and c.

$a = 1, b = {}^{-}5, c = 4$

STEP 2: Create a list of all factor pairs for $a \bullet c$ ($1 \bullet 4 = \mathbf{4}$) and determine which pair *add* to make b ($^{-}\mathbf{5}$).

The factor pair for **4** that adds to make $^{-}\mathbf{5}$ is $^{-}1$ and $^{-}4$.



STEP 3: Finally, take the factors that satisfy **Step 2** and fill them into the form:

$(x + \_\_\_\_)(x + \_\_\_\_)$

$(x - \underline{1})(x - \underline{4})$

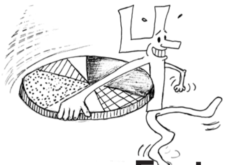
So, $x^2 - 5x + 4$ factored is $(x - 1)(x - 4)$.

***Try this*: Factor the following polynomial.**

1) $x^2 - 11x + 30$

$\underline{^{-}5}$ & ____ *multiply* to make **30** and *add* to make $^{-}\mathbf{11}$.

So, $x^2 - 11x + 30 = (x - \_\_\_\_)(x - \_\_\_\_)$.



• Factoring Quadratic Polynomials •

Factor the following polynomials.

1) $y^2 - y - 2$

____ & ____ *multiply* to make $^{-}\mathbf{2}$ and *add* to make $^{-}\mathbf{1}$.

So, $y^2 - y - 2 = (y + \_\_\_\_)(y - \_\_\_\_)$.

2) $x^2 + 3x - 18$

____ & ____ *multiply* to make $^{-}\mathbf{18}$ and *add* to make $\mathbf{3}$.

So, $x^2 + 3x - 18 = (x - \_\_\_\_)(x + \_\_\_\_)$.

---

3) $m^2 + 4m - 32$

= ____________________

4) $x^2 - 6x - 27$

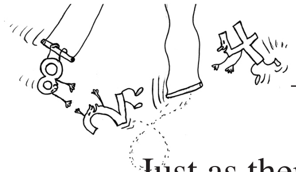
= ____________________

5) $y^2 - 10y + 24$

= ____________________

6) $n^2 + 14n + 24$

= ____________________



• Prime Polynomials • ☆

Just as there are *prime* numbers, there are also *prime* polynomials.

A QUADRATIC POLYNOMIAL is *prime* when there are no two integers that *multiply* to make $a \bullet c$ and *add* to make b.

EXAMPLE: Factor $x^2 + 5x + 7$.

Neither of the factor pairs that *multiply* to make **7** ($a \bullet c$) also *add* to make **5** (b).

FACTORS OF $a \bullet c$	SUM OF b	
1 & 7	8	✗
$^{-}1$ & $^{-}7$	$^{-}8$	✗

So, $x^2 + 5x + 7$ is *prime*.

Try these: **Factor the following polynomials. If the polynomial is prime, write "prime" on the answer line.**

1) $x^2 - 8x - 15$

= ______________________

2) $y^2 + 3y - 28$

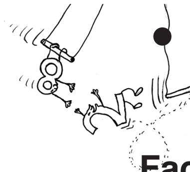
= ______________________

3) $x^2 + 16x + 55$

= ______________________

4) $z^2 - 2z + 9$

= ______________________



• Factoring Quadratic Polynomials *Practice* • ☆

Factor the following polynomials. If the polynomial is prime, write "prime" on the answer line.

1) $k^2 + 5k - 36$

= ______________________

2) $n^2 - 17n + 38$

= ______________________

3) $x^2 - 16x + 56$

= ______________________

4) $z^2 + 14z + 45$

= ______________________

5) $x^2 - 12x - 13$

= ______________________

6) $y^2 + 9y - 10$

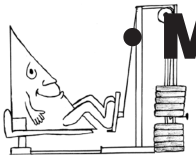
= ______________________

7) $h^2 - 5h + 66$

= ______________________

8) $x^2 - 8x - 48$

= ______________________



• Mastery Check: Factoring Quadratics •

Factor the following polynomials.

1) $y^2 - 10y - 24$

$= \underline{\hspace{6cm}}$

2) $x^2 + 13x - 30$

$= \underline{\hspace{6cm}}$

3) $x^2 - 5x + 6$

$= \underline{\hspace{6cm}}$

4) $v^2 + 18v + 77$

$= \underline{\hspace{6cm}}$

5) $z^2 - 14z + 49$

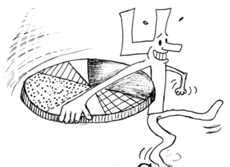
$= \underline{\hspace{6cm}}$

6) $x^2 + 10x + 25$

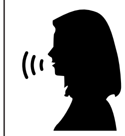
$= \underline{\hspace{6cm}}$

Challenge:

7) $x^2 - 32xy - 144y^2 = \underline{\hspace{6cm}}$



• Factoring by Grouping •



Sometimes, not all terms in a **POLYNOMIAL** have a common factor. In this case when factoring, we must look for groups of terms that have common factors.

EXAMPLE: Factor $ax + 3a + 2x + 6$.

Steps to Solve:

STEP 1: First, group the first two and last two terms so that each pair has common factors. ax and $3a$ have a in common, while $2x$ and 6 have 2 in common.

$$(ax + 3a) + (2x + 6)$$

STEP 2: Factor each pair by pulling out their respective **GCF**s.

$$a(x + 3) + 2(x + 3)$$

STEP 3: If possible, factor again. Both terms have a factor of $(x + 3)$ in common. So, factor it out and leave what is left of each term in parentheses.

$$a(x + 3) + 2(x + 3)$$
$$= (x + 3)(a + 2)$$

So, $ax + 3a + 2x + 6$ factored is $(x + 3)(a + 2)$.

We can check our answer by multiplying the resulting binomials, $(x + 3)$ and $(a + 2)$, together. We should have the original **POLYNOMIAL**.

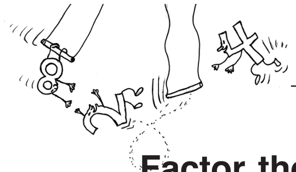
$$(x + 3)(a + 2) = ax + 3a + 2x + 6 \checkmark$$

Try this: **Factor the following polynomial by grouping. Check your answers using the DISTRIBUTIVE PROPERTY.**

1) $cy + c + 5y + 5 =$ (__cy__ + __c__) + (____ + ____)

= __c__ (__y__ + __1__) + ____ (____ + ____)
GCF (1) GCF (2)

= (__y__ + __1__) (____ + ____)



• Factoring by Grouping •

Factor the following polynomials by grouping. Check your answers using the DISTRIBUTIVE PROPERTY.

1) $vw + v^2 + 5w + 5v = (\underline{\;vw\;} + \underline{\;v^2\;}) + (\underline{\qquad} + \underline{\qquad})$

$= \underset{\text{GCF (1)}}{\underline{\;v\;}}(\underline{\;w\;} + \underline{\;v\;}) + \underset{\text{GCF (2)}}{\underline{\qquad}}(\underline{\qquad} + \underline{\qquad})$

$= (\underline{\qquad} + \underline{\qquad})(\underline{\qquad} + \underline{\qquad})$

2) $3x^3 + 6x^2 + 2x + 4 = (\underline{\qquad} + \underline{\qquad}) + (\underline{\qquad} + \underline{\qquad})$

$= \underset{\text{GCF (1)}}{\underline{\qquad}}(\underline{\qquad} + \underline{\qquad}) + \underset{\text{GCF (2)}}{\underline{\qquad}}(\underline{\qquad} + \underline{\qquad})$

$= (\underline{\qquad} + \underline{\qquad})(\underline{\qquad} + \underline{\qquad})$

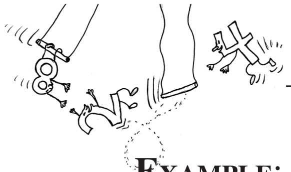
---

3) $xy + 2y + 7x + 14 =$ ____________________

4) $ab + bc + ad + cd =$ ____________________

5) $s^2 + s + as + a =$ ____________________

6) $z^3 + z^2 + 8z + 8 =$ ____________________



• Factoring by Grouping •

EXAMPLE: Factor $mz - 5m - 3z + 15$.

$$(mz - 5m) + ({}^-3z + 15)$$
$$= m(z - 5) + 3({}^-z + 5)$$

To get a common factor, we must also pull a $^-1$ out of the second term.

$$= m(z - 5) + {}^-3(z - 5)$$
$$= (z - 5)(m - 3)$$

So, $mz - 5m - 3z + 15$ factored is $(z - 5)(m - 3)$.

Factor the following polynomials by grouping.

1) $ax - bx - 2a + 2b =$ ________________________

2) $qr - 5q + 2r - 10 =$ ________________________

3) $dx - d + 6x - 6 =$ ________________________

4) $10xz + 5xy - 2z - y =$ ________________________

• Factoring by Grouping •

Factor the following polynomials by grouping.

1) $q^2r + 4q^2 + 4r + 16$ = ________________________

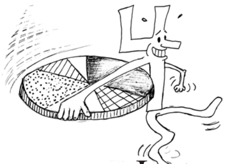
2) $xy + 5x + 5y + 25$ = ________________________

3) $rx - r - qx + q$ = ________________________

4) $fg - 2f - 2g + 4$ = ________________________

5) $aw + 7w + a + 7$ = ________________________

6) $ay + a - y - 1$ = ________________________



• Prime Polynomials •

☆

Just as there are *prime* numbers, there are also *prime* POLYNOMIALS.

For factoring by grouping, this means there is no way to group the terms so that the original POLYNOMIAL can be written as the product of two or more POLYNOMIALS of a lesser degree.

EXAMPLE: Factor $cx + cg + 5h + 5g$.

$$\begin{aligned} &(cx + cg) + (5h + 5g) \\ = \; &c(x + g) + 5(h + g) \end{aligned}$$

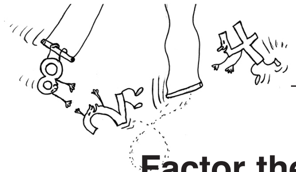
These two terms do not have any common factors, so $cx + cg + 5h + 5g$ is *prime*.

NOTE: No matter how we rearrange the terms, the POLYNOMIAL cannot be factored by grouping.

Try these: **Factor the following polynomials if possible. If the polynomial is prime, write "prime" on the answer line.**

1) $hk + hm - 10k - 20m =$ ________________________

2) $a^2 - 2ab - 2a + 4b =$ ________________________



• Factoring by Grouping •

Factor the following polynomials by grouping if possible. If the polynomial is prime, write "prime" on the answer line.

1) $a^3 + 2a^2 + a + 2 =$ ________________________

2) $2xy + 10x + y + 5 =$ ________________________

3) $wx - x + 4w - 5 =$ ________________________

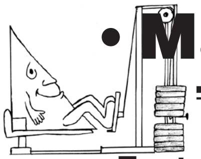
4) $3xy - y - 3x + 1 =$ ________________________

5) $ab - ac + 2b - 2c =$ ________________________

6) $yz + 5z^2 + 11y + 55z =$ ________________________

MC

Bypass



• Mastery Check: Factoring by Grouping • ☆

Factor the following polynomials by grouping if possible. If the polynomial is prime, write "prime" on the answer line.

1) $6rs + 3r + 4s + 2 =$ ________________________

2) $pq - 3qr - 6p + 18r =$ ________________________

3) $6ax - 8ay + 9bx - 12by =$ ________________________

4) $14vw + 6v - 24w - 9 =$ ________________________

5) $8st - 2s + 4t^2 - t =$ ________________________

Challenge:

6) $15a^2y^4 + 54by^5 - 5a^2xz^3 - 18bxyz^3 =$ ________________________

• Difference of Perfect Squares •

Direct Teaching

The QUADRATIC POLYNOMIALS in the form $m^2 - n^2$ are called the DIFFERENCE OF PERFECT SQUARES. When factored, there is a pattern that we can point out. This pattern can be written as a formula and it occurs in *any* polynomial that is the DIFFERENCE OF PERFECT SQUARES.

DIFFERENCE OF PERFECT SQUARES FORMULA
$m^2 - n^2 = (m + n)(m - n)$

EXAMPLE: Factor $x^2 - 81$.

Since x^2 and 81 are both perfect squares and they are written as the difference of each other, we can use the **DIFFERENCE OF PERFECT SQUARES FORMULA**.

$$x^2 - 81$$

$$= (\underline{\ x\ })^2 - (\underline{\ 9\ })^2$$

$$= (\underline{\ x\ } + \underline{\ 9\ })(\underline{\ x\ } - \underline{\ 9\ })$$

So, $x^2 - 81$ factored is $(x + 9)(x - 9)$.

Try these: **Factor the following polynomials using the DIFFERENCE OF PERFECT SQUARES FORMULA.**

1) $x^2 - 49$

$= (\underline{\ x\ })^2 - (\underline{\qquad})^2$

$= (\underline{\ x\ } + \underline{\qquad})(\underline{\ x\ } - \underline{\qquad})$

2) $y^2 - 64$

$= (\underline{\qquad})^2 - (\underline{\qquad})^2$

$= (\underline{\qquad} + \underline{\qquad})(\underline{\qquad} - \underline{\qquad})$

• Difference of Perfect Squares •

DIFFERENCE OF PERFECT SQUARES FORMULA

$$m^2 - n^2 = (m + n)(m - n)$$

EXAMPLE: Factor $4y^2 - 25$.

$$4y^2 - 25$$

$$= (\underline{\ 2y\ })^2 - (\underline{\ 5\ })^2$$

$$= (\underline{\ 2y\ } + \underline{\ 5\ })(\underline{\ 2y\ } - \underline{\ 5\ })$$

So, $4y^2 - 25$ factored is $(2y + 5)(2y - 5)$.

Factor the following DIFFERENCE OF PERFECT SQUARES polynomials.

1) $x^2 - 121$

$= (\_\_\_\_\_)^2 - (\_\_\_\_\_)^2$

$= (\_\_\_\_\_ + \_\_\_\_\_)(\_\_\_\_\_ - \_\_\_\_\_)$

2) $9z^2 - 1$

$= (\_\_\_\_\_)^2 - (\_\_\_\_\_)^2$

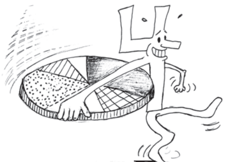
$= (\_\_\_\_\_ + \_\_\_\_\_)(\_\_\_\_\_ - \_\_\_\_\_)$

3) $v^2 - 100$

= ____________________

4) $25w^2 - 36$

= ____________________



• Difference of Perfect Squares •

EXAMPLE: Factor $16v^6 - 9w^2$.

$$16v^6 - 9w^2$$

$$= (\underline{\;4v^3\;})^2 - (\underline{\;3w\;})^2$$

$$= (\underline{\;4v^3\;} + \underline{\;3w\;})(\underline{\;4v^3\;} - \underline{\;3w\;})$$

So, $16v^6 - 9w^2$ factored is $(4v^3 + 3w)(4v^3 - 3w)$.

Try these: **Factor the following DIFFERENCE OF PERFECT SQUARES polynomials.**

1) $9z^8 - 1$

$= (\underline{\qquad})^2 - (\underline{\qquad})^2$

$= (\underline{\qquad} + \underline{\qquad})(\underline{\qquad} - \underline{\qquad})$

2) $121x^4 - 36y^2$

$= (\underline{\qquad})^2 - (\underline{\qquad})^2$

$= (\underline{\qquad} + \underline{\qquad})(\underline{\qquad} - \underline{\qquad})$

3) $25x^2 - 36$

= ______________________

4) $64y^6 - 9z^4$

= ______________________

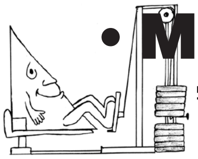
5) $^{-}9y^2 + 1$

= ______________________

6) $(x + y)^2 - 25$

= ______________________

Extending Knowledge



• Mastery Check: Difference of Perfect Squares •

Completely factor the following DIFFERENCE OF PERFECT SQUARES polynomials.

1) $z^2 - 169$

= ____________________________

2) $49w^4 - 1$

= ____________________________

3) $x^2y^2 - 9$

= ____________________________

4) $25y^2 - 36$

= ____________________________

5) $q^8 - 4$

= ____________________________

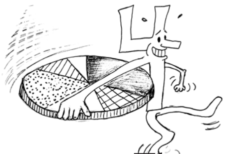
6) $9y^2z^6 - 64$

= ____________________________

Challenge:

7) (☺^{2} – ☹^{2})^2 – (☆^{2} – ◊^{2})^2

= ____



• Absolute Value Equations •

The ABSOLUTE VALUE of a number is its **distance from zero** on the number line. ABSOLUTE VALUE can be used in algebraic equations.

EXAMPLE 1: $|x| = 2$

You can think of the equation as asking the question, "What numbers are 2 units away from **zero**? Its two solutions can be shown on a number line:

Both these values, $^-2$ and 2, satisfy the condition of being 2 units from 0. We can write the answer as:

$$x = \{^-2, 2\}$$

Check: $|x| = |{}^-2| = 2$ $\quad\quad$ $|x| = |2| = 2$

EXAMPLE 2: $|x| = {}^-7$

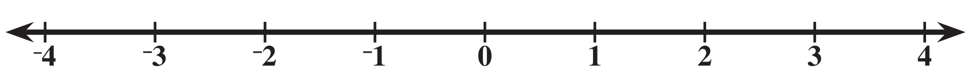
The ABSOLUTE VALUE, or ***distance*** from **zero**, can never be **negative**.

Since there are **no** values for x which will make an ABSOLUTE VALUE, or ***distance***, **negative**, this equation has no solution.

$$x = \{\ \}$$

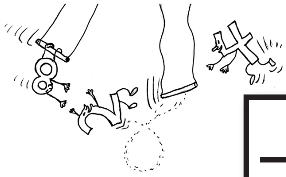
Try these:

1) Graph the solutions to the following equation on a number line: $|x| = 3$.



$x =$ ____________

2) TRUE or FALSE: $|{}^-x| = {}^-4$ has a solution. ______________



• Absolute Value Equations •

Absolute Value

If the **ABSOLUTE VALUE** of $\boldsymbol{x}$ equals a, where $a \geq 0$, then $\boldsymbol{x} = \pm a$.

$$|x| = a \longrightarrow x = \pm a, \text{ for } a \geq 0$$

To solve an **ABSOLUTE VALUE EQUATION**, the absolute value **expression** must be **isolated** on one side of the equation. Then, examine the two cases that have the same distance from zero.

EXAMPLE: $|x + 6| = 10$

Another way to view this equation:
What **values** of $\boldsymbol{x}$ make the expression $\boldsymbol{x} + 6$ have a distance of 10 from zero?

CASE 1: $x + 6 = 10$ **CASE 2**: $x + 6 = {}^{-}10$

Solve **both** cases for $\boldsymbol{x}$:

$$\begin{array}{rcl} x + 6 & = & 10 \\ -6 & & -6 \\ \hline x & = & \mathbf{4} \end{array} \qquad \begin{array}{rcl} x + 6 & = & {}^{-}10 \\ -6 & & -6 \\ \hline x & = & \mathbf{{}^{-}16} \end{array}$$

Check the solutions:

$$\begin{array}{rcl} |(\mathbf{4}) + 6| & \stackrel{?}{=} & 10 \\ |10| & \stackrel{?}{=} & 10 \\ 10 & \stackrel{\checkmark}{=} & 10 \end{array} \qquad \begin{array}{rcl} |(\mathbf{{}^{-}16}) + 6| & \stackrel{?}{=} & 10 \\ |{}^{-}10| & \stackrel{?}{=} & 10 \\ 10 & \stackrel{\checkmark}{=} & 10 \end{array}$$

So, the equation has **two** solutions: $x = \{\mathbf{4}, \mathbf{{}^{-}16}\}$.

Try these: **In each exercise, solve the absolute value equation.**

1) $|x - 3| = 7$

$x =$ ____________

2) $|x + 4| = 1$

$x =$ ____________

• Solving Absolute Value Equations •

Before an **ABSOLUTE VALUE EQUATION** can be solved, the absolute value expression must be **isolated** on one side of the equation. The absolute value bars are a grouping symbol for the expression inside them, like parentheses or brackets.

$2\lvert x + 8\rvert + 4 = 10$	$2\lvert x + 8\rvert = 6$ $\lvert x + 8\rvert = 3$	$x + 8 = 3$ $x + 8 = {}^{-}3$
The absolute value expression must be **isolated** before solving.	Subtract 4 from both sides and divide both sides by 2 to isolate the absolute value expression.	The absolute value equation can now be solved.

***Try these*:** **In each exercise, solve the absolute value equation.**

1) $2\lvert 4x - 1\rvert + 3 = 11$

$x =$ ____________

2) ${}^{-}\lvert 6x + 2\rvert - 5 = {}^{-}13$

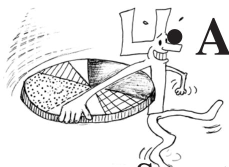
$x =$ ____________

3) $3\lvert x + 8\rvert + 4 = 10$

$x =$ ____________

4) $5\lvert 2x - 6\rvert = 20$

$x =$ ____________



• Absolute Value Equations – Extraneous Solutions •

Sometimes, one or more solutions that you find for an absolute value equation will form an invalid equation when substituted into the original equation. These are EXTRANEOUS SOLUTIONS and they are **not** solutions to the equation.

It is important to **always** check your solutions in the original equation to ensure you avoid including any EXTRANEOUS SOLUTIONS.

$\lvert 2x+1\rvert = 4x$	$2x + 1 = 4x$ $^-2x = {^-1}$ $x = \frac{1}{2}$ $2x + 1 = {^-4x}$ $6x = {^-1}$ $x = -\frac{1}{6}$	$\left\lvert 2\left(\frac{1}{2}\right) + 1\right\rvert \overset{?}{=} 4\left(\frac{1}{2}\right)$ $2 \overset{\checkmark}{=} 2$ $\left\lvert 2\left(-\frac{1}{6}\right) + 1\right\rvert \overset{?}{=} 4\left(-\frac{1}{6}\right)$ $\left\lvert \frac{2}{3}\right\rvert \overset{\times}{=} -\frac{2}{3}$
An absolute value equation with a variable on both sides of the equation.	Solve the remaining absolute value equation.	The value $x = -\frac{1}{6}$ results in an invalid solution. Therefore, it is EXTRANEOUS.

To explore why we may sometimes find EXTRANEOUS SOLUTIONS, examine the original equation in the example above:

Since it is an absolute value equation, the expression on the right side ($4x$) must be greater than or equal to zero: $4x \geq 0$.

Dividing both sides by 4 to solve the inequality results in $x \geq 0$.

Since the found solution $x = -\frac{1}{6}$ does not satisfy $x \geq 0$, it is an EXTRANEOUS SOLUTION!

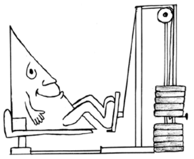
***Try these*:** **In each exercise, solve the absolute value equation. Check for extraneous solutions. If there are no solutions, write "no solutions."**

1) $\lvert 3x\rvert = x - 12$

$x =$ ______________________

2) $3\lvert 2x - 5\rvert = {^-9x}$

$x =$ ______________________



• Mastery Check: Absolute Value Equations •

In each exercise, solve the absolute value equation. Check for extraneous solutions. If there are no solutions, write "no solutions."

1) $|4x - 3| + 4 = 9$

$x =$ ______________________

2) $|{-2x} + 5| = 7$

$x =$ ______________________

3) $|5x - 3| = -4$

$x =$ ______________________

4) $4|3x - 6| + 5 = 17$

$x =$ ______________________

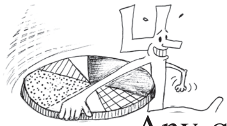
Challenge:

5) $|x + 2| = |3x|$

$x =$ ______________________

6) $|{-2x} - 4| = |8x + 6|$

$x =$ ______________________



• The Quadratic Formula •

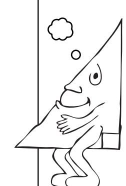
Any quadratic equation in **standard form** ($ax^2 + bx + c = 0$, where $a \neq 0$) can be solved using the QUADRATIC FORMULA.

> **The Quadratic Formula**
>
> The solutions to a quadratic equation $ax^2 + bx + c = 0$ are
>
> $$x = \frac{^-b \pm \sqrt{b^2 - 4ac}}{2a}\text{, where } a \neq 0.$$

The ± symbol in the QUADRATIC FORMULA indicates there are two solutions:

$$x = \frac{^-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{^-b - \sqrt{b^2 - 4ac}}{2a}.$$

When possible, use factoring instead of the **QUADRATIC FORMULA** to solve a quadratic equation.



EXAMPLE: Solve the quadratic equation using the QUADRATIC FORMULA.

$$x^2 + 3x + 2 = 0$$

This quadratic equation can be solved by factoring: $x^2 + 3x + 2 = (x + 1)(x + 2) = 0$.
By factoring, we see that the solutions to the equation are $x = {}^-1$ and $x = {}^-2$.

To use the QUADRATIC FORMULA method instead:

STEP 1: First, identify a, b, and c in the quadratic equation: $a = 1$, $b = 3$, $c = 2$.
STEP 2: Then, substitute a, b, and c into the **QUADRATIC FORMULA** to solve for x.

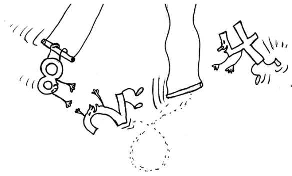
$$\text{So, } x = \frac{^-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{^-3 \pm \sqrt{3^2 - 4(1)(2)}}{2(1)} = \frac{^-3 \pm \sqrt{1}}{2} = \frac{^-3 \pm 1}{2}.$$

The solutions to the equation are $x = \frac{^-3 + 1}{2} = \mathbf{^-1}$ and $x = \frac{^-3 - 1}{2} = \mathbf{^-2}$.

Try this: **Solve the quadratic equation using the QUADRATIC FORMULA.**

1) $2x^2 + 7x + 6 = 0$ $\qquad a =$ ______ , $b =$ ______ , $c =$ ______

$x =$ ____________ or ____________



• The Quadratic Formula •

The QUADRATIC FORMULA is useful to solve a quadratic equation with a polynomial that cannot be factored.

$x^2 - 8x + 13 = 0$	$x = \dfrac{^{-}(^{-}8) \pm \sqrt{(^{-}8)^2 - 4(1)(13)}}{2(1)}$	$x = \dfrac{8 \pm \sqrt{12}}{2} = \dfrac{8 \pm 2\sqrt{3}}{2}$ $= 4 \pm \sqrt{3}$ So, $x = 4 + \sqrt{3}$ or $4 - \sqrt{3}$.
$a = 1$, $b = {^{-}8}$, and $c = 13$	Substitute a, b, and c into the QUADRATIC FORMULA.	Simplify and find the solutions to the equation.

1) Write the QUADRATIC FORMULA.

$x =$ ______________________ , where $a \neq 0$

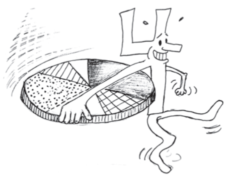
In each exercise, solve the quadratic equation using the QUADRATIC FORMULA.

2) $2x^2 + 6x + 3 = 0$

$x =$ _____________ or _____________

3) $^{-}4x^2 - 3x + 2 = 0$

$x =$ _____________ or _____________



• The Discriminant: $b^2 - 4ac$ •

In the quadratic formula, the expression inside the radical sign ($b^2 - 4ac$) is called the DISCRIMINANT. The value of the DISCRIMINANT determines the nature of the solutions to a quadratic equation.

Value of the discriminant	Number of real solutions to the quadratic equation	Example
$b^2 - 4ac > 0$	Two real solutions	$x^2 - 5x + 2 = 0$ $x = \frac{^-(^-5) \pm \sqrt{(^-5)^2 - 4(1)(2)}}{2(1)}$ So, $x = \frac{5}{2} + \frac{\sqrt{17}}{2}$ or $\frac{5}{2} - \frac{\sqrt{17}}{2}$.
$b^2 - 4ac = 0$	One real solution, repeated	$9x^2 + 6x + 1 = 0$ $x = \frac{^-6 \pm \sqrt{6^2 - 4(9)(1)}}{2(9)} = \frac{^-6 \pm \mathbf{0}}{18} = -\frac{1}{3}$
$b^2 - 4ac < 0$	No real solutions	$2x^2 - 3x + 4 = 0$ Since $b^2 - 4ac = {^-23}$ is negative and the square root of a negative number has no real value, $\sqrt{b^2 - 4ac}$ has no real value. Therefore, there are no real solutions.

***Try these*: Calculate the discriminant to find the number of real solutions.**

1) $^-3x^2 + 2x - 4 = 0$

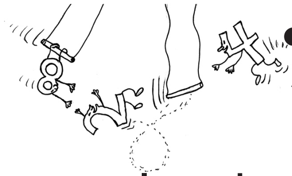
$b^2 - 4ac =$ ________

Number of real solutions: _____

2) $^-4x^2 - 2x + 3 = 0$

$b^2 - 4ac =$ ________

Number of real solutions: _____



• The Quadratic Formula *Practice* •

In each exercise, solve the quadratic equation using the QUADRATIC FORMULA. If there are no real solutions, write "no real solutions."

1) $^{-}4x^2 + 5x + 3 = 0$

$x =$ ____________________

2) $^{-}3x^2 + 4x - 1 = 0$

$x =$ ____________________

3) $9x^2 - 6x + 1 = 0$

$x =$ ____________________

4) $x^2 - 4x - 3 = 0$

$x =$ ____________________

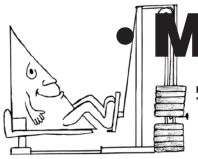
5) $3x^2 + x + 6 = 0$

$x =$ ____________________

6) $x^2 - 7x - 1 = 4$

Extending Knowledge

$x =$ ____________________



• Mastery Check: The Quadratic Formula •

In each exercise, solve the quadratic equation using the QUADRATIC FORMULA. If there are no real solutions, write "no real solutions."

1) $2x^2 - 5x - 13 = 0$

$x =$ ____________________

2) $^{-}4x^2 + 5x - 1 = 0$

$x =$ ____________________

3) $x^2 + 6x + 9 = 0$

$x =$ ____________________

4) $^{-}7x^2 + 3x - 2 = 0$

$x =$ ____________________

Challenge: Solve the cubic equation.

5) $x^3 - 5x^2 + 3x = 0$

$x =$ ____________________

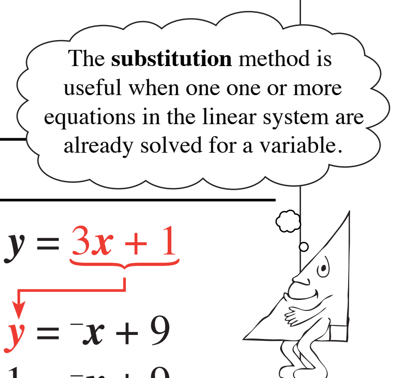
• Solving a System of Linear Equations by Substitution •

For a SYSTEM OF LINEAR EQUATIONS (or LINEAR SYSTEM) with two variables and two equations, a **solution** to the linear system is a solution to **both** equations. When the linear system is graphed, a point where the equation lines intersect is a **solution** to the linear system.

Another method to solve a linear system with two variables and two equations is **substitution**. In this method, one equation will be used to substitute an expression for a variable in another equation. This results in an equation with a single variable that can be solved. After the first variable is solved, it can then be substituted into one of the original equations to solve for the remaining variable.

EXAMPLE: Solve the linear system by substitution. Check your answer.

$$\begin{cases} ① \ y = 3x + 1 \\ ② \ y = {}^{-}x + 9 \end{cases}$$



Steps to Solve

STEP 1:
Since both equations are solved for y, substitute the equivalent expression for y from equation ① in place of y in equation ②.

① $y = \underline{3x + 1}$

② $y = {}^{-}x + 9$

$3x + 1 = {}^{-}x + 9$

STEP 2:
Solve the remaining equation for x.

$3x + 1 = {}^{-}x + 9$

$4x + 1 = 9$

$x = 2$

STEP 3:
Substitute the value of x into equation ① or ② to find the value of y and complete the solution.

① $y = 3(2) + 1$

$y = 7$

So, $(x, y) = (2, 7)$.

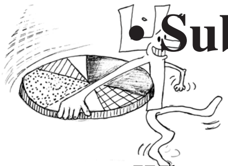
Verify the solution by substituting the value of x and y into each original equation.

① $7 \stackrel{?}{=} 3(2) + 1 \stackrel{\checkmark}{=} 7$ ② $7 \stackrel{?}{=} {}^{-}(2) + 9 \stackrel{\checkmark}{=} 7$

Try this: **Solve the linear system by substitution. Check your answer.**

1) $\begin{cases} ① \ y = 2x - 6 \\ ② \ y = 3x - 7 \end{cases}$

$(x, y) =$ ___________



• Substitution Method – Different Forms of Equations • ☆

When one equation in a LINEAR SYSTEM is solved for a variable and the other equation is not, we can still use the **substitution** method. In this case, substitute the expression for the solved variable into the other equation.

EXAMPLE: Solve the linear system by substitution. Check your answer.

$$\begin{cases} ① \ y = {}^{-}2x + 1 \\ ② \ 3x + 2y = 4 \end{cases}$$

Steps to Solve

STEP 1:
Equation ① is solved for y, so substitute the equivalent expression (${}^{-}2x + 1$) for y in equation ②.

$$① \ y = \underbrace{{}^{-}2x + 1}$$
$$② \ 3x + 2y = 4$$
$$3x + 2({}^{-}2x + 1) = 4$$

STEP 2:
Solve the remaining equation for x.

$$\begin{aligned} 3x + 2({}^{-}2x + 1) &= 4 \\ 3x - 4x + 2 &= 4 \\ {}^{-}x + 2 &= 4 \\ x &= {}^{-}2 \end{aligned}$$

STEP 3:
Substitute the value of x into equation ① or ② to find the value of y and complete the solution.

$$① \ y = {}^{-}2({}^{-}2) + 1$$
$$y = 5$$

So, $(x, y) = ({}^{-}2, 5)$.

Verify the solution by substituting the value of x and y into each original equation.

$$① \ 5 \stackrel{?}{=} {}^{-}2({}^{-}2) + 1 \stackrel{\checkmark}{=} 5 \qquad ② \ 3({}^{-}2) + 2(5) = {}^{-}6 + 10 \stackrel{\checkmark}{=} 4$$

***Try this*: Solve the linear system by substitution. Check your answer.**

1) $\begin{cases} ① \ x = {}^{-}y - 3 \\ ② \ {}^{-}2x - y = 2 \end{cases}$

$(x, y) =$ __________

• Solving for a Variable Before Substitution • ☆

Substitution is a useful method to solve a **LINEAR SYSTEM** when one equation can be solved for a variable quickly, such as when one variable has a coefficient of 1.

EXAMPLE: Solve the linear system by substitution. Verify the solution.

$$\begin{cases} ① \ x + 4y = 10 \\ ② \ 2x + 3y = 15 \end{cases}$$

Steps to Solve

STEP 1:
In equation ①, the coefficient of x is 1. Since it can be solved for x in one step, solve equation ① for x.

$$① \ x + 4y = 10$$
$$x = {}^{-}4y + 10$$

STEP 2:
Substitute the equivalent expression ($^{-}4y + 10$) for x in equation ②.

$$② \ 2x + 3y = 15$$
$$2({}^{-}4y + 10) + 3y = 15$$

STEP 3:
Solve the remaining equation for y.

$$2({}^{-}4y + 10) + 3y = 15$$
$${}^{-}8y + 20 + 3y = 15$$
$${}^{-}5y + 20 = 15$$
$${}^{-}5y = {}^{-}5$$
$$y = \mathbf{1}$$

STEP 4:
Substitute the value of y into equation ① or ② to find the value of x and complete the solution.

$$① \ x + 4(\mathbf{1}) = 10$$
$$x + 4 = 10$$
$$x = \mathbf{6}$$
So, $(x, y) = (6, 1)$.

Verify the solution by substituting the value of x and y into each original equation.

$① \ 6 + 4(1) = 6 + 4 \overset{\checkmark}{=} 10$ $\qquad$ $② \ 2(6) + 3(1) = 12 + 3 \overset{\checkmark}{=} 15$

***Try this*:** **Solve the linear system by substitution. Check your answer.**

1) $\begin{cases} ① \ 3x + y = 1 \\ ② \ 4x - 2y = 18 \end{cases}$

$(x, y) =$ ___________

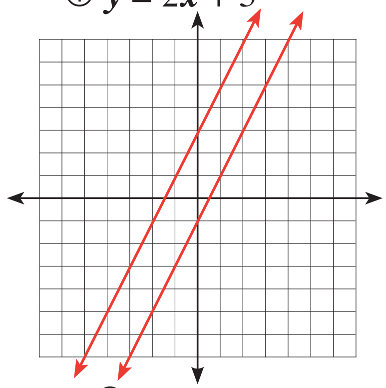
• Inconsistent System – No Solution Exists • ☆

Observe the result when attempting to solve the following linear system:

$$\begin{cases} ① \; y = 2x + 3 \\ ② \; y - 2x = {}^{-}1 \end{cases} \qquad \begin{array}{l} ① \; y = 2x + 3 \\ ② \; y - 2x = {}^{-}1 \end{array} \qquad (2x + 3) - 2x = {}^{-}1 \qquad 3 \overset{\times}{=} {}^{-}1$$

The final equation is **impossible**. When the linear system is graphed, we can see two **parallel** lines that do not intersect. Therefore, there is **no solution**.

① $y = 2x + 3$



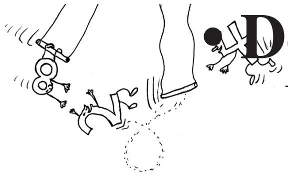
② $y - 2x = {}^{-}1$

A linear system that has no solution is an **INCONSISTENT SYSTEM**. When attempting to use the substitution method to solve an **INCONSISTENT SYSTEM**, an impossible equation will result.

Try this: **Use substitution to solve the linear system. If an impossible equation results, write the impossible equation and "no solutions."**

1) $\begin{cases} ① \; 6x + 2y = 1 \\ ② \; y = {}^{-}3x + 4 \end{cases}$

____

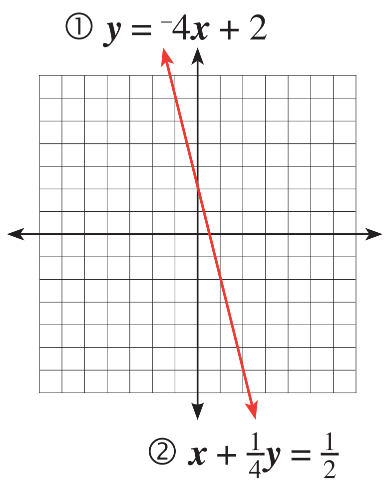


• Dependent System – Infinite Solutions • ☆

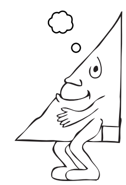
Observe the result when attempting to solve the following linear system:

$$\begin{cases} ① \; y = {}^{-}4x + 2 \\ ② \; x + \frac{1}{4}y = \frac{1}{2} \end{cases} \qquad \begin{aligned} ① \; y &= \underbrace{{}^{-}4x + 2} \\ ② \; x + \frac{1}{4}y &= \frac{1}{2} \end{aligned} \qquad x + \frac{1}{4}({}^{-}4x + 2) = \frac{1}{2} \qquad \frac{1}{2} = \frac{1}{2}$$

In the final equation, the variables have been eliminated and the equation is valid. When the linear system is graphed, we can see that each equation is represented by the **same** line. Therefore, there are **infinite solutions** – every point on the line is a solution to both equations.



Equations that share the same line are **equivalent equations**! You can transform one equation into the other equation algebraically. Here, one equation is a "multiple" of the other equation.



A linear system that has infinite solutions is a DEPENDENT SYSTEM. When attempting to use the substitution method to solve a DEPENDENT SYSTEM, a valid equation without variables will result.

Try this: **Use substitution to solve the linear system. If a true equation results without any variables, write the true equation and "infinite solutions."**

1) $\begin{cases} ① \; y = {}^{-}2x \\ ② \; 4x + 2y = 0 \end{cases}$

____

• Solving a System of Linear Equations by Substitution • ☆

Solve the linear system by substitution. Check your answer. If there are no solutions, write "no solutions." If there are infinite solutions, write "infinite solutions."

1) $\begin{cases} ① \; y = 3x + 5 \\ ② \; y = x - 3 \end{cases}$

____

2) $\begin{cases} ① \; y = {}^{-}3x + 1 \\ ② \; 6x + 2y = 5 \end{cases}$

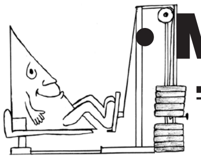
____

3) $\begin{cases} ① \; {}^{-}4x + y = {}^{-}8 \\ ② \; 2x + 3y = 18 \end{cases}$

____

4) $\begin{cases} ① \; {}^{-}x + y = 2 \\ ② \; \frac{1}{2}x - \frac{1}{4}y = 4 \end{cases}$

____



• Mastery Check: Substitution Method •

Solve the linear system by substitution. Check your answer. If there are no solutions, write "no solutions." If there are infinite solutions, write "infinite solutions."

1) $\begin{cases} ① \; y = {}^{-}3x + 13 \\ ② \; 4x + 3y = 29 \end{cases}$

____

2) $\begin{cases} ① \; x - 4y = 7 \\ ② \; 5x - 2y = {}^{-}1 \end{cases}$

____

3) $\begin{cases} ① \; y = {}^{-}2x + 3 \\ ② \; y = {}^{-}2x - 1 \end{cases}$

____

Challenge:

4) $\begin{cases} ① \; x + 2y - z = 1 \\ ② \; y = z - x \\ ③ \; 2x - y + 3z = 12 \end{cases}$

____

• Solving a System of Linear Equations by Elimination • ☆

Direct Teaching

One method of solving a linear system is **elimination**. In the elimination method, the equations are added together or subtracted to eliminate a variable and obtain a new equation in the remaining variable. After solving this equation for the variable, it can be substituted into one of the original equations to solve for the other variable.

In the linear system below, the coefficients of y, 3 and $^{-}3$, are **opposite**, as their sum is zero ($3 + {}^{-}3 = 0$). When the coefficients of a variable are **opposite**, **add** the equations together to eliminate the variable.

Example: Solve the linear system by elimination. Check your answer.

$$\begin{cases} ① \ 4x + 3y = 15 \\ ② \ 5x - 3y = 12 \end{cases}$$

Steps to Solve

Step 1:
Since the coefficients of y are opposite, add equations ① and ② together to eliminate y.

$$\begin{array}{rl} ① & 4x + 3y = 15 \\ + \ ② & (5x - 3y = 12) \\ \hline & 9x \qquad = 27 \end{array}$$

Step 2:
Solve the remaining equation for x.

$$9x = 27$$
$$x = 3$$

Step 3:
Substitute the value of x into equation ① or ② to find the value of y and complete the solution.

$$① \ 4(3) + 3y = 15$$
$$12 + 3y = 15$$
$$y = 1$$

So, $(x, y) = (3, 1)$.

Verify the solution by substituting the value of x and y into each original equation.

$① \ 4(3) + 3(1) = 12 + 3 \overset{\checkmark}{=} 15$ $\qquad$ $② \ 5(3) - 3(1) = 15 - 3 \overset{\checkmark}{=} 12$

Try this: **Solve the linear system by elimination. Check your answer.**

1) $\begin{cases} ① \ 2x - 5y = {}^{-}11 \\ ② \ {}^{-}2x + y = 7 \end{cases}$

$(x, y) =$ __________

Elimination Method – Same Coefficients

The elimination method can also be used to solve a linear system by eliminating a variable that has the **same coefficient** in each equation.

When the coefficient of the variable is the **same** in each equation, **subtract** one equation from the other to eliminate the variable.

EXAMPLE: Solve the linear system by elimination. Check your answer.

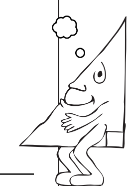
$$\begin{cases} ① \ 6x + 9y = 3 \\ ② \ 6x - 2y = 36 \end{cases}$$

Remember, subtracting $^{-}2y$ is the same as adding $2y$.

Steps to Solve

STEP 1:
Since x has the same coefficient in each equation, subtract equation ② from ① to eliminate x.

$$\begin{array}{r} ① \ 6x + 9y = 3 \\ - \ ② \ (6x - 2y = 36) \\ \hline 11y = {}^{-}33 \end{array}$$



STEP 2:
Solve the remaining equation for y.

$$11y = {}^{-}33$$
$$y = {}^{-}3$$

STEP 3:
Substitute the value of y into equation ① or ② to find the value of x and complete the solution.

$$① \ 6x + 9({}^{-}3) = 3$$
$$6x - 27 = 3$$
$$x = 5$$

So, $(x, y) = (5, {}^{-}3)$.

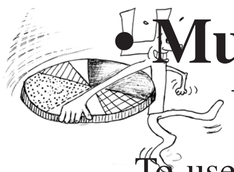
Verify the solution by substituting the value of x and y into each original equation.

$① \ 6(5) + 9({}^{-}3) = 30 - 27 \stackrel{\checkmark}{=} 3$ $\qquad$ $② \ 6(5) - 2({}^{-}3) = 30 + 6 \stackrel{\checkmark}{=} 36$

Try this: **Solve the linear system by elimination. Check your answer.**

1) $\begin{cases} ① \ 2x + 5y = 2 \\ ② \ 4x + 5y = {}^{-}6 \end{cases}$

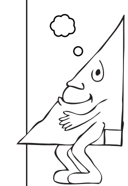
$(x, y) =$ ____________



• Multiplying an Equation Before Elimination • ☆

To use the elimination method, it is sometimes necessary to first **multiply** one or more equations to have variables with the **same** or **opposite** coefficients in the linear system.

You can also use **elimination** by multiplying equation ① by $^-3$ and adding the equations to eliminate x, if you prefer.



EXAMPLE: Solve the linear system by elimination.

$$\begin{cases} ① \ 6x + 7y = 9 \\ ② \ 2x + 5y = 11 \end{cases}$$

A linear system that can be solved using the elimination method.

$$3(2x + 5y) = 3(11)$$
$$6x + 15y = 33$$

Multiply both sides of equation ② by 3 to change the coefficient of x *from 2 to 6.*

$$① \ 6x + 7y = 9$$
$$6x + 15y = 33$$

With the same coefficient of x *in both equations,* x *can be eliminated by subtraction.*

Try these: **Solve the linear system by elimination. Check your answer.**

1) $\begin{cases} ① \ {}^-4x + 3y = 1 \\ ② \ 5x - 6y = {}^-8 \end{cases}$

$(x, y) =$ ___________

2) $\begin{cases} ① \ 2x + 5y = 18 \\ ② \ 4x + y = 0 \end{cases}$

$(x, y) =$ ___________

3) $\begin{cases} ① \ 4x - 2y = {}^-6 \\ ② \ {}^-8x + 3y = 7 \end{cases}$

$(x, y) =$ ___________

• Multiplying Both Equations Before Elimination • ☆

It can be useful to first rewrite **both** equations using multiplication before using the elimination method to solve a linear system.

EXAMPLE: Solve the linear system by elimination. Check your answer.

$$\begin{cases} ① \ 3x + 4y = 11 \\ ② \ 4x + 5y = 14 \end{cases}$$

Steps to Solve

STEP 1:
Multiply each equation so that the coefficients of x are both 12, the LCM of 3 and 4.

$$4(3x + 4y) = 4(11)$$
$$3(4x + 5y) = 3(14)$$

STEP 2:
Now that x has the same coefficient in both equations, subtract the new equation ② from the new equation ① to eliminate x and solve for y.

$$\begin{array}{rr} & 12x + 16y = 44 \\ - & (12x + 15y = 42) \\ \hline & y = \mathbf{2} \end{array}$$

STEP 3:
Substitute the value of y into equation ① or ② to find the value of x and complete the solution.

$$3x + 4(\mathbf{2}) = 11$$
$$3x + 8 = 11$$
$$x = \mathbf{1}$$

So, $(x, y) = (1, 2)$.

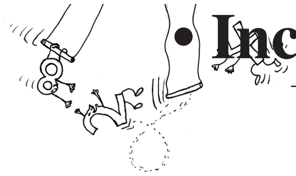
Verify the solution by substituting the value of x and y into each original equation.

① $3(\mathbf{1}) + 4(\mathbf{2}) = 3 + 8 \overset{\checkmark}{\stackrel{?}{=}} 11$ ② $4(\mathbf{1}) + 5(\mathbf{2}) = 4 + 10 \overset{\checkmark}{\stackrel{?}{=}} 14$

Try this: **Solve the linear system by elimination. Check your answer.**

1) $\begin{cases} ① \ 2x - 3y = {}^-6 \\ ② \ 3x - 4y = {}^-7 \end{cases}$

$(x, y) =$ __________

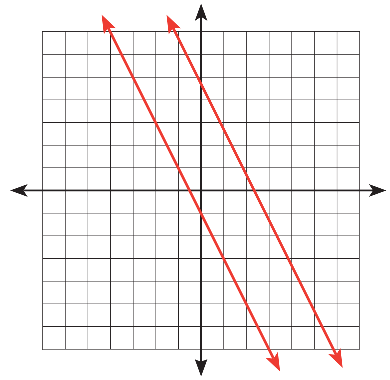


• Inconsistent System – No Solution Exists • ☆

Observe the result when attempting to solve the following linear system:

$$\begin{cases} ① \ 4x + 2y = {}^{-}2 \\ ② \ 2x + y = 5 \end{cases} \qquad \begin{aligned} 2(2x + y) &= 2(5) \\ 4x + 2y &= 10 \end{aligned} \qquad \begin{array}{r} ① \ 4x + 2y = {}^{-}2 \\ - \ \ (4x + 2y = 10) \\ \hline 0 \overset{?}{=} {}^{-}12 \end{array} \qquad 0 \overset{\times}{=} {}^{-}12$$

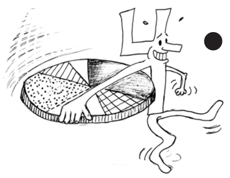
The final equation is **impossible**. When the linear system is graphed, we can see two **parallel** lines that do not intersect. Therefore, there is **no solution**.



A linear system that has no solution is an **INCONSISTENT SYSTEM**. When attempting to use the elimination method to solve an **INCONSISTENT SYSTEM**, an impossible equation will result.

Try this: **Use the elimination method to solve the linear system. If an impossible equation results, write the impossible equation and "no solutions."**

1) $\begin{cases} ① \ 3x - 2y = 1 \\ ② \ 6x - 4y = 8 \end{cases}$

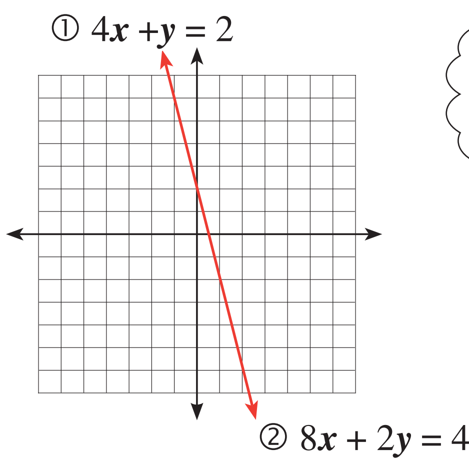


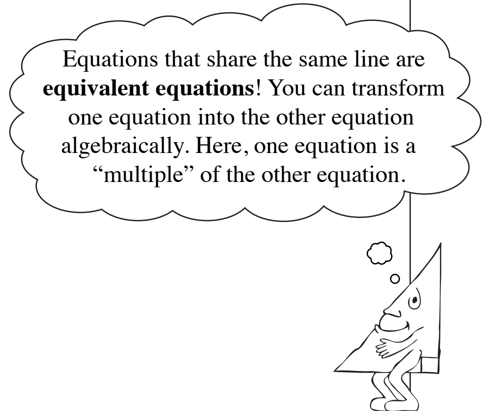
• Dependent System – Infinite Solutions • ☆

Observe the result when attempting to solve the following linear system:

$$\begin{cases} ① \ 4x + y = 2 \\ ② \ 8x + 2y = 4 \end{cases} \qquad \begin{aligned} 2(4x + y) &= 2(2) \\ 8x + 2y &= 4 \end{aligned} \qquad \begin{array}{r} 8x + 2y = 4 \\ -\ (8x + 2y = 4) \\ \hline 0 \overset{?}{=} 0 \end{array} \qquad 0 = 0$$

In the final equation, the variables have been eliminated and the equation is valid. When the linear system is graphed, we can see that each equation is represented by the **same** line. Therefore, there are **infinite solutions** – every point on the line is a solution to both equations.





A linear system that has infinite solutions is a **DEPENDENT SYSTEM**. When attempting to use the elimination method to solve a **DEPENDENT SYSTEM**, a valid equation without variables will result.

Try this: **Use the elimination method to solve the linear system. If a true equation results without any variables, write the equation and "infinite solutions."**

1) $\begin{cases} ① \ 3x - 4y = {}^{-}1 \\ ② \ 6x - 8y = {}^{-}2 \end{cases}$

____

• Solving a System of Linear Equations by Elimination • ☆

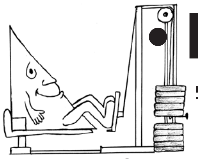
Solve the linear system by elimination. Check your answer. If there are no solutions or infinite solutions, write "no/infinite solutions."

1) $$\begin{cases} \textcircled{1}\ 2x - 5y = 1 \\ \textcircled{2}\ 3x + 4y = 13 \end{cases}$$

2) $$\begin{cases} \textcircled{1}\ 2x + 4y = 16 \\ \textcircled{2}\ x + 2y = 5 \end{cases}$$

3) $$\begin{cases} \textcircled{1}\ \frac{1}{2}x - \frac{3}{4}y = 0 \\ \textcircled{2}\ \frac{1}{2}x + \frac{1}{4}y = 4 \end{cases}$$

4) $$\begin{cases} \textcircled{1}\ 5x - 6y = 14 \\ \textcircled{2}\ {}^{-}5x + 9y = {}^{-}11 \end{cases}$$



• **Mastery Check:** Elimination Method • ☆

Solve the linear system by elimination. Check your answer. If there are no solutions or infinite solutions, write "no/infinite solutions."

1) $\begin{cases} ① \ 2x + 3y = 8 \\ ② \ 5x - 6y = {}^{-}7 \end{cases}$

____

2) $\begin{cases} ① \ x + 4y = 9 \\ ② \ 2x + 8y = 5 \end{cases}$

____

3) $\begin{cases} ① \ 5x - 7y = {}^{-}11 \\ ② \ {}^{-}5x + 3y = {}^{-}1 \end{cases}$

____

Challenge:

4) $\begin{cases} ① \ x - y + z = 9 \\ ② \ x + y - z = 3 \\ ③ \ {}^{-}x - y - z = {}^{-}7 \end{cases}$

____

• What Are the Parts? What Is the Whole? •

☆

"The whole equals the sum of its parts."

PART + PART = WHOLE

1) 12 pounds of white sand is mixed with 4 pounds of black sand.

The first "part" in the mixture is __________ pounds of white sand.

The second "part" in the mixture is __________ pounds of black sand.

The "whole" mixture is __________ pounds of sand.

__________% of the sand mixture is white.

__________% of the sand mixture is black.

---

2) A quarter of a liter of bubble bath liquid is mixed into a bath. The water and bubble bath liquid mixed together make 100 liters.

The first "part" in the mixture is __________ liter of bubble bath liquid.

The second "part" in the mixture is __________ liters of water.

The "whole" mixture is __________ liters.

__________% of the mixture is bubble bath liquid.

__________% of the mixture is water.

---

3) 1 ounce of pigment is mixed into 15 ounces of paint medium.

The first "part" in the mixture is __________ ounce of pigment.

The second "part" in the mixture is __________ ounces of medium.

The "whole" mixture is __________ ounces of paint.

__________% of the paint is pigment.

__________% of the paint is medium.

• The Value of Each Part •

We can find the value of a part by multiplying the whole by an applicable rate or percentage.

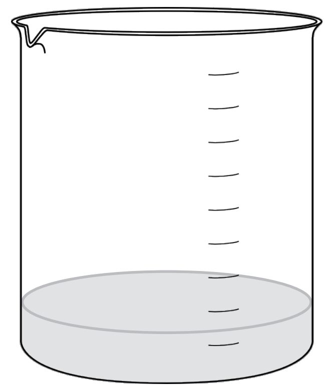
Example: The beaker pictured below holds 20 fluid ounces. If it is 15% filled with acid, how many fluid ounces of acid are in the beaker?

To answer this question, we multiply the capacity of the beaker by the percentage to find the part of the beaker that is full.

"Whole • Rate = Part"

20 fluid ounces • 15% = 3 fluid ounces

There are 3 fluid ounces of acid in the beaker.



Try these: **Find the quantity.**

1) How much pure juice is there in 2 gallons of 12.5% juice drink?

____

2) How much pure saline is in 4 liters of a 16% saline solution?

____

3) How much fat is there in 200 gallons of 2% milk?

____

4) How many ounces of pigment are there in a gallon of paint that is 12.5% pigment? (A gallon is 128 fluid ounces.)

____

• Wholes & Parts of Mixtures •

A mixture of nuts contains 5 pounds of peanuts that cost $4 per pound and 3 pounds of cashews that cost $8 per pound, resulting in a mixture of nuts that is worth $5.50 per pound.

1) What are the "parts" in this scenario? What is the "whole"?

Part: ____

Part: ____

Whole: ____

2) What is the total cost of the peanuts?

____

3) What is the total cost of the cashews?

____

4) What is the total weight of the entire mixture? What is its total cost?

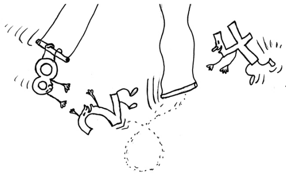
There are ______________ pounds of nuts costing $ ______________.

5) Write an equation that represents the scenario:

$$\underbrace{\_\_\_\_\ \text{LB} \bullet \$\_\_\_\_\ \text{PER LB}}_{\textbf{PEANUTS}} + \underbrace{\_\_\_\_\ \text{LB} \bullet \$\_\_\_\_\ \text{PER LB}}_{\textbf{CASHEWS}} = \underbrace{\_\_\_\_\ \text{LB} \bullet \$\_\_\_\_\ \text{PER LB}}_{\textbf{MIXTURE}}$$

6) **True** or **false**: The total weight on one side of the equation is equal to the total weight on the other side of the equation.

7) **True** or **false**: The total cost on one side of the equation is equal to the total cost on the other side of the equation.



• Wholes & Parts of Mixtures •

Example: Raisins cost $4.00 per pound and almonds cost $7.00 per pound. Twenty pounds of raisins are mixed with 10 pounds of almonds. What is the cost per pound of the resulting mixture?

First, we need to identify the wholes and the parts. One part is 20 pounds of raisins, the other part is 10 pounds of almonds, and the whole is the sum of the raisins and the almonds.

The value of each part is its amount multiplied by its cost per pound.

20 pounds • $4.00 per pound + 10 pounds • $7.00 per pound

The same goes for the whole. The total amount of mixture is 30 pounds, and the cost per pound is unknown.

30 pounds • ***x*** per pound

Now, since we know that the whole is equal to the sum of its parts, we can write a full equation to solve.

20 LB • $4.00 PER LB + 10 LB • $7.00 PER LB = 30 LB • ***x*** PER LB

$80 + 70 = 30x$

$x = \$5.00$

The cost per pound of the resulting mixture is $5.00.

Try this:

1) Soda water costs 50¢ per quart. Juice concentrate costs $1.20 per quart. What is the cost per quart of a fizzy juice drink made of 4 quarts of soda water and 1 quart of juice concentrate?

_____ QT • $_____ PER QT + _____ QT • $_____ PER QT = _____ QT • $_____ PER QT

____

• Wholes & Parts of Mixtures •

The equation below represents what happens when we mix 2 liters of 10% juice with 6 liters of 30% juice.

2 LITERS • 10% JUICE + 6 LITERS • 30% JUICE = _____ LITERS • _____% JUICE

1) What are the “parts” in this equation? What is the “whole”?

Part: ____

Part: ____

Whole: ____

2) What is the amount of pure juice in 2 liters of 10% juice?

____

3) What is the amount of pure juice in 6 liters of 30% juice?

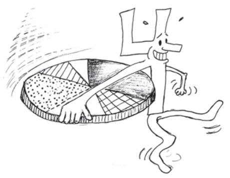
____

4) What is the total amount of pure juice in the whole mixture?

____

5) What percentage of the resulting mixture is pure juice?

____



• Wholes & Parts of Mixtures •

EXAMPLE: How much of a 50% salt solution should be mixed with 10 liters of a 10% salt solution in order to create a mixture that is 25% salt?

In this scenario, the amount of one part of the mixture is unknown. The total amount must therefore be the sum of the known amount and the unknown amount (*i.e.*, $10 + x$).

$$10 \text{ liters} \bullet 10\% \text{ salt} + x \text{ liters} \bullet 50\% \text{ salt} = (10 + x) \text{ liters} \bullet 25\% \text{ salt}$$

$$10 \bullet 0.1 + x \bullet 0.5 = (10 + x) \bullet 0.25$$

$$1 + 0.5x = 2.5 + 0.25x$$

$$0.25x = 1.5$$

$$x = 6 \text{ liters}$$

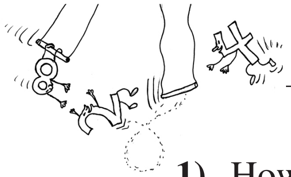
Six liters of 50% salt solution must be added to 10 liters of 10% salt solution to create a mixture that is 25% salt.

Try this:

1) How many liters of 30% acid must be added to 2 liters of 15% acid to result in a mixture that is 25% acid.

_____ L • _____% ACID + _____ L • _____% ACID = (_____+_____) L • _____% ACID

____



• Mixture Problem Practice •

☆

1) How many gallons of 50% grape juice must be added to 16 gallons of 25% grape juice to result in a mixture that is 40% juice?

____

2) How many ounces of 45% pigmented paint must be added to 12 ounces of 25% pigmented paint to result in a paint mixture that is $37\frac{1}{2}\%$ pigment?

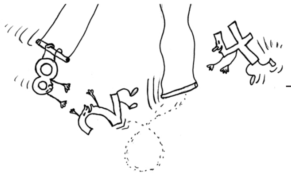
____

3) Margaret mixes the remaining 6 cups of a 25% concentrated detergent with the remaining 4 cups of an $87\frac{1}{2}\%$ concentrated detergent. What is the concentration of the leftover detergent mixture?

____

4) How much 100% active ingredient powder would you need to add to 4 mL of multivitamin powder that is 20% active ingredient powder in order to strengthen the mixture to 50% active ingredient powder?

____



• Two Unknowns, One Variable •

When there are two unknown quantities in a scenario, we can either write a system in two variables or represent both unknowns using the same variable within two different expressions.

Example: An amount of 50% juice is mixed with an amount of 80% juice to make 20 cups of 75% juice. Write two expressions to represent the unknown volume of each kind of juice.

When we eventually write an equation to solve for the unknown volumes of each ingredient, we will use x in place of the unknown volume of 50% juice.

Amount of 50% juice: x

Now, let's use the same variable to write an expression that represents the other unknown quantity. In any whole-and-parts scenario, each part is equal to the whole minus the other parts. The whole in this scenario is 20 cups, and the other part is x.

Amount of 80% juice: $20 - x$

Try this: **Write two expressions to represent the two unknowns in each scenario below. Use only one variable.**

1) An amount of 5% acid is mixed with an amount of 40% acid to make 10 liters of a 20% acid solution.

Amount of 5% acid: __________

Amount of 40% acid: __________

Amount of 20% acid: __________

• Mixture Problems •

Example: How much of a 50% juice and an 80% juice should be mixed together to make 20 liters of a 75% juice?

In this scenario, both amounts of the parts are unknown, but the amount of the whole mixture is known. So, we know that the sum of the parts must be 20 liters.

First, let's solve for the amount of the 50% juice that goes into the mixture. We'll call that amount $\boldsymbol{x}$. Since the whole is the sum of the parts, that means that the other amount must be $\boldsymbol{x}$ less than 20 liters.

$$\boldsymbol{x} \text{ liters} \bullet 50\% \text{ juice} + (20 - \boldsymbol{x}) \text{ liters} \bullet 80\% \text{ juice}$$

Now we can write an equation to solve for the unknown amount of 50% juice.

$$\boldsymbol{x} \bullet 0.5 + (20 - \boldsymbol{x}) \bullet 0.8 = 20 \bullet 0.75$$

$$0.5\boldsymbol{x} + 16 - 0.8\boldsymbol{x} = 15$$

$$^{-}0.3\boldsymbol{x} = {}^{-}1$$

$$\boldsymbol{x} = 3\tfrac{1}{3} \text{ liters}$$

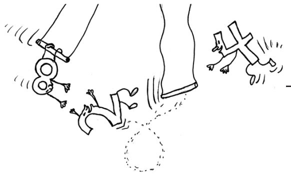
So, if $3\frac{1}{3}$ liters of 50% juice go into the mixture, then $16\frac{2}{3}$ liters of 80% juice must go into the mixture to make a total of 20 liters of 75% juice.

Try this:

1) How much 30% acetone and 80% acetone should be mixed together to have 50 cups of 60% acetone?

____

____



• Mixture Problem Practice •

1) 24-karat gold is made of 100% pure gold. How much 12-karat gold and how much 6-karat gold should be smelted together to create 20 grams of metal that is exactly 40% gold?

____

____

2) How many cups of 25% dye must be added to 16 cups of $6\frac{1}{4}$% dye to result in a mixture that is $12\frac{1}{2}$% dye?

____

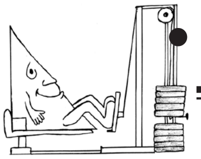
3) How much 75% concentrated cleaning solution and 100% concentrated cleaning solution should be mixed together to have 16 cups of 80% concentrated cleaning solution?

____

____

4) A glass of chocolate milk was made from a mixture of 2 fluid ounces of 5% chocolate syrup and 8 fluid ounces of 20% chocolate syrup. What is the percentage of milk in the glass of chocolate milk?

____



• Mastery Check: Mixture Problems •

1) What is the concentration of acid in a mixture of 10 cups of 50% acid and 14 cups of 20% acid?

____

2) How many milliliters of paste that is 25% recycled plastic must be added to 40 milliliters of paste that is 10% recycled plastic to result in a mixture that is 20% recycled plastic?

____

3) There is a 10% salt solution and a 30% salt solution. How much of each is needed to make 10 liters of a mixture that is 25% salt?

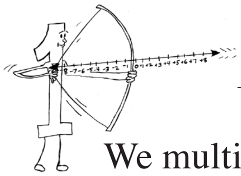
____

____

Challenge:

4) 5 liters of 10% salt solution are mixed with 15 liters of 30% salt. How much water must evaporate from the mixture so that it is 40% salt solution?

____



• Multiply Rational Expressions •

Direct Teaching

We multiply rational expressions the same way we multiply numeric fractions: we multiply the numerators, then multiply the denominators. If the expressions are polynomials, we write them in factored form $[(x + a)(x + b)]$, if possible, instead of using the distributive property to write them in standard form $[ax^2 + bx + c]$.

Example: Simplify $\dfrac{x - 1}{x^2 - 5x + 6} \bullet \dfrac{x^2 + 6x + 8}{x^2 - 11x + 30}$.

First, let's write the polynomials in factored form.

$$\frac{x - 1}{x^2 - 5x + 6} \bullet \frac{x^2 + 6x + 8}{x^2 - 11x + 30} = \frac{(x - 1)}{(x - 2)(x - 3)} \bullet \frac{(x + 2)(x + 4)}{(x - 5)(x - 6)}$$

After writing the terms in factored form, we combine the numerators and denominators.

$$\frac{(x - 1)}{(x - 2)(x - 3)} \bullet \frac{(x + 2)(x + 4)}{(x - 5)(x - 6)} = \frac{(x - 1)(x + 2)(x + 4)}{(x - 2)(x - 3)(x - 5)(x - 6)}$$

We leave our solution in factored form. That is considered simplified form.

Try these: **Simplify the following.**

1) $\dfrac{x - 10}{x^2 - 9x + 8} \bullet \dfrac{x^2 - 12x - 13}{x + 7}$

FACTORED: $\dfrac{x - 10}{(x - 1)(x - 8)} \bullet$ ______________

SIMPLIFIED:

____

2) $\dfrac{x^2 + 3x + 2}{x + 15} \bullet \dfrac{x - 4}{3x^2 - x - 2}$

FACTORED: ______________ • ______________

SIMPLIFIED:

____

• Divide Rational Expressions •

Direct Teaching

We divide rational expressions the same way we divide fractions: we invert and multiply. If the expressions are polynomials, it is beneficial to write the polynomials in factored form.

Example: Simplify $\dfrac{x^2+6x+5}{x^2-6x+8} \div \dfrac{x+9}{2x^2+7x+3}$.

First let's write the polynomials in factored form.

$$\frac{x^2+6x+5}{x^2-6x+8} \div \frac{x+9}{2x^2+7x+3} = \frac{(x+1)(x+5)}{(x-2)(x-4)} \div \frac{(x+9)}{(2x+1)(x+3)}$$

After writing the terms in factored form, we invert and multiply.

$$\frac{(x+1)(x+5)}{(x-2)(x-4)} \div \frac{(x+9)}{(2x+1)(x+3)} = \frac{(x+1)(x+5)}{(x-2)(x-4)} \bullet \frac{(2x+1)(x+3)}{(x+9)}$$

$$= \frac{(x+1)(x+5)(2x+1)(x+3)}{(x-2)(x-4)(x+9)}$$

So, $\dfrac{x^2+6x+5}{x^2-6x+8} \div \dfrac{x+9}{2x^2+7x+3}$ simplifies to $\dfrac{(x+1)(x+5)(2x+1)(x+3)}{(x-2)(x-4)(x+9)}$.

Try these: **Simplify the following.**

1) $\dfrac{x^2-5x+4}{x^2-25} \div \dfrac{x^2+10x}{x+9}$

FACTORED:

$\dfrac{(x-1)(x-4)}{(x-5)(x+5)} \div$ ____________

INVERT AND MULTIPLY:

$\dfrac{(x-1)(x-4)}{(x-5)(x+5)} \bullet$ ____________

SIMPLIFIED:

____

2) $\dfrac{x^2+7x+12}{x+15} \div \dfrac{x-4}{2x^2+x-3}$

FACTORED:

____________ $\div$ ____________

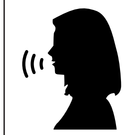
INVERT AND MULTIPLY:

____________ $\bullet$ ____________

SIMPLIFIED:

____

• Simplifying Rational Expressions •



Direct Teaching

When we have a product or a quotient of rational expressions, we can simplify further if there is a term in the numerator that matches a term in the denominator.

Example: Simplify $\dfrac{x-3}{x^2+x-12} \bullet \dfrac{x^2-16}{x^2-9x+20}$.

First, let's write the polynomials in factored form and then multiply.

$$\frac{x-3}{x^2+x-12} \bullet \frac{x^2-16}{x^2-9x+20} = \frac{(x-3)}{(x-3)(x+4)} \bullet \frac{(x-4)(x+4)}{(x-4)(x-5)}$$

As long as the same term is on the top and bottom of a rational expression, it can be reduced.

$$\frac{\cancel{x-3}}{\cancel{(x-3)}\cancel{(x+4)}} \bullet \frac{\cancel{(x-4)}\cancel{(x+4)}}{\cancel{(x-4)}(x-5)} = \frac{1}{(x-5)}$$

Try these: **Simplify the following fractions.**

1) $\dfrac{x^2-3x-4}{x^2-16} \bullet \dfrac{x^2+4x}{x-2}$

FACTORED: $\dfrac{(x+1)(x-4)}{(x-4)(x+4)} \bullet$ ____________

SIMPLIFIED:

____

2) $\dfrac{x^2-7x+12}{x+1} \div \dfrac{x-4}{x^2-2x-3}$

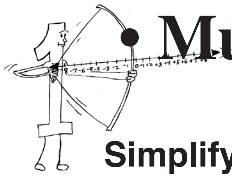
FACTORED: ____________ $\div$ ____________

INVERT AND MULTIPLY:

____________ $\bullet$ ____________

SIMPLIFIED:

____



• Multiply and Divide Rational Expressions • ☆

Simplify the following fractions.

1) $\dfrac{x^2 - 3x - 4}{x^2 - 16} \bullet \dfrac{x^2 + 4x}{x - 2}$

FACTORED: ____________ • ____________

SIMPLIFIED:

____

2) $\dfrac{p^2}{2p^2 + 5p + 3} \div \dfrac{7p^2 - 3p}{4p^2 + 4p + 1}$

SIMPLIFIED:

____

3) $\dfrac{x^2 + 5x + 6}{x^2 + 5x - 6} \bullet \dfrac{x^2 - 5x + 6}{x^2 - 5x - 6}$

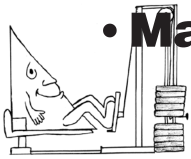
SIMPLIFIED:

____

4) $\dfrac{c^3 - 13c^2 + 42c}{c - 6} \div \dfrac{3c^2 + c}{9c^2 + 12c + 4}$

SIMPLIFIED:

____



• **Mastery Check:** Multiply and Divide Rational Expressions •

☆

Simplify the following fractions.

1) $\dfrac{x^2 + 7x + 10}{x} \bullet \dfrac{x^2}{x^2 + 2x - 15}$

SIMPLIFIED: ______________________

2) $\dfrac{x^2 + 8x + 15}{x^2 - 7x + 12} \div \dfrac{x + 5}{x^2 - 3x - 18}$

SIMPLIFIED: ______________________

3) $\dfrac{2n^2 + 3n + 1}{n^2 - 3n + 2} \div \dfrac{6n^2 + 5n + 1}{n^2 - 5n + 4}$

SIMPLIFIED: ______________________

Challenge:

4) $\dfrac{z^3 - 27}{z^2 - z - 6} \div \dfrac{z^4 - 16}{z^4 - 5z^2 + 4}$

SIMPLIFIED: ______________________

• Adding and Subtracting Square Roots •

The Law of SAMEness

Direct Teaching

For the purposes of this pack, assume all variables are positive.

The Law of SAMEness

We can only add and subtract things that are of the same denomination, things that have the same name.

Recall that the radicand is the value *inside* the radical. The radicand is the *name* of the term, so we can only add and subtract square roots when they have the same radicand value.

Example 1: Simplify $2\sqrt{11} + 7\sqrt{11}$.

We can add the square roots because they have radicands of the same value.

$2\sqrt{11} + 7\sqrt{11} =$

2 **square roots of eleven** + 7 **square roots of eleven** =

9 **square roots of eleven** =

$9\sqrt{11}$

If the radicands are simplified and not the same, then we cannot combine the square roots.

Example 2: Simplify $6\sqrt{13} - 2\sqrt{17}$.

$\sqrt{13}$ and $\sqrt{17}$ are not the same radicands, which means they do not have the same name. So we cannot combine the radicals. Therefore, the expression cannot be simplified.

***Try these*: Simplify. Rewrite the expression if it is already simplified.**

1) $8\sqrt{15} + 6\sqrt{15} =$ ____________

2) $18\sqrt{5} - 11\sqrt{5} =$ ____________

3) $13\sqrt{7} - 7\sqrt{13} =$ ____________

4) $11\sqrt{x} + 8\sqrt{x} =$ ____________

• Simplifying Square Roots Before Combining • ☆

We may have to simplify one or more radicals first to see if we can add or subtract radicals.

EXAMPLE: Simplify $3\sqrt{10} + 6\sqrt{40}$.

The radicands $\sqrt{10}$ and $\sqrt{40}$ are not the same. But $6\sqrt{40}$ can be simplified. Let's see if we can add the square roots after simplifying.

$$3\sqrt{10} + 6\sqrt{40} =$$

$$3\sqrt{10} + 12\sqrt{10} =$$

$$15\sqrt{10}$$

So, $3\sqrt{10} + 6\sqrt{40}$ simplifies to $15\sqrt{10}$.

***Try these*: Simplify.**

1) $2\sqrt{32} + 7\sqrt{18} =$

______ + ______ =

____

2) $4\sqrt{12} - 10\sqrt{27} =$

______ − ______ =

____

3) $10\sqrt{20} + 12\sqrt{45} =$

______ + ______ =

____

4) $y\sqrt{300} + y\sqrt{75} =$

______ + ______ =

____

• Adding and Subtracting Square Roots •

Simplify.

1) $\sqrt{5} + \sqrt{20} =$ ____________

2) $\sqrt{81} - 3\sqrt{9} =$ ________________

3) $\sqrt{16h^3} - 2h\sqrt{8h} =$ ____________

4) $4n\sqrt{21} + 2\sqrt{42n} =$ ____________

5) $3w\sqrt{w^3} - 5\sqrt{w^5} =$ ____________

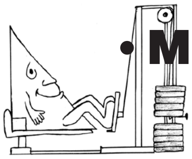
6) $10\sqrt{25} + 4\sqrt{12} =$ ____________

7) $10q\sqrt{18p^3q} + 2p\sqrt{32pq^3} =$ __________

8) $\sqrt{8h} - 9\sqrt{2h} =$ ______________

9) $10ab\sqrt{500ab} - 8\sqrt{20a^3b^3} =$ _________

10) $3y\sqrt{4xy} + \sqrt{25xy^3} =$ ___________



• Mastery Check: Adding and Subtracting Square Roots •

Simplify. Rewrite the expression if it is already simplified.

1) $\sqrt{75} + \sqrt{48} =$ ____________

2) $3\sqrt{7} - 2\sqrt{14} =$ ____________

3) $2\sqrt{16} - 3\sqrt{9} =$ ____________

4) $\sqrt{75} - \sqrt{50} =$ ____________

5) $\sqrt{7k} + \sqrt{28k} =$ ____________

6) $x\sqrt{22} + \sqrt{99x^2} =$ ____________

7) $y\sqrt{x^5y^3} + x\sqrt{x^3y^5} =$ ____________

8) $n\sqrt{300} + \sqrt{432n^2} =$ ____________

Challenge:

9) $\sqrt{27} + \sqrt{75} + \sqrt{48} =$ ____________

10) $\sqrt{\frac{8}{9}} + \sqrt{\frac{32}{25}} =$ ____________

• Point–Slope Form •

Direct Teaching

An equation in POINT–SLOPE FORM is written as:

$$\boxed{y - y_1 = m(x - x_1)}$$

where m is the SLOPE of the line that goes through the point (x_1, y_1).

If we are given the slope and a point on a line, we can use POINT–SLOPE FORM to find the equation of the line.

EXAMPLE: Find the equation of the line in POINT–SLOPE FORM with a slope of 2 that goes through the point $(3, {}^{-}1)$.

We are given the slope $m = 2$ and the point on the line, $(3, {}^{-}1)$. So, $x_1 = 3$ and $y_1 = {}^{-}1$. Let's use these values to write the equation of the line in POINT–SLOPE FORM:

$$y - y_1 = m(x - x_1)$$
$$y - ({}^{-}1) = 2(x - 3)$$
$$y + 1 = 2(x - 3)$$

So, the equation of the line with a slope of 2 that goes through the point $(3, {}^{-}1)$ in POINT–SLOPE FORM is $y + 1 = 2(x - 3)$.

Try this: **Write the equation of the line in POINT–SLOPE FORM.**

1) Write the equation of the line with a slope of 4 that goes through the point $({}^{-}5, {}^{-}2)$.

____

• Point–Slope Form •

☆

Write the equation of each line in POINT–SLOPE FORM.

1) Write the equation of the line with a slope of 2 that goes through the point $(7, {}^{-}3)$.

____

2) Write the equation of the line with a slope of $\frac{1}{4}$ that goes through the point $({}^{-}1, 0)$.

____

3) Write the equation of the line with a slope of ${}^{-}9$ that goes through the point $(12, 15)$.

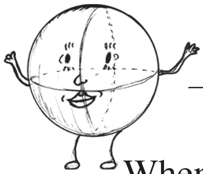
____

4) Write the equation of the line with a slope of $-\frac{1}{3}$ that goes through the point $(\frac{1}{2}, -\frac{2}{5})$.

____

5) Write the equation of the line with a slope of m that goes through the point (x_1, y_1).

____



• Point–Slope to Slope–Intercept Form •

When given an equation of a line in **POINT–SLOPE FORM**, we can rewrite the equation in **SLOPE–INTERCEPT FORM** by solving the equation for y.

EXAMPLE: Write $y + 1 = 2(x - 3)$ in **SLOPE–INTERCEPT FORM**.

If we solve for y, we can find the equation of the line in **SLOPE–INTERCEPT FORM**.

$$y + 1 = 2(x - 3)$$

$$y + 1 = 2x - 6$$

$$y = 2x - 7$$

So, $y + 1 = 2(x - 3)$ in **SLOPE–INTERCEPT FORM** is $y = 2x - 7$.

Try these: **Write the equation of each line in SLOPE–INTERCEPT FORM.**

1) $y + 7 = 3(x - 2)$

____

2) $y - 3 = \frac{1}{2}(x + 8)$

____

3) $y - 1 = -\frac{1}{3}(x - 7)$

____

4) $y + \frac{3}{4} = \frac{1}{4}(x - 13)$

____

5) $y + 9 = {}^{-}4(x - \frac{1}{8})$

____

6) $y - y_1 = m(x - x_1)$

____

• Finding the Equation of a Line •

1) Write the equation of the line with a slope of $^-6$ that goes through the point $(9, {}^-4)$.

POINT–SLOPE FORM: ______________________________

SLOPE–INTERCEPT FORM: ______________________________

2) Write the equation of the line with a slope of $\frac{3}{5}$ that goes through the point $({}^-1, 8)$.

POINT–SLOPE FORM: ______________________________

SLOPE–INTERCEPT FORM: ______________________________

3) Write the equation of the line with a slope of m that goes through the point $(0, b)$. Assume m is a positive integer.

Extending Knowledge

POINT–SLOPE FORM: ______________________________

SLOPE–INTERCEPT FORM: ______________________________

STANDARD FORM: ______________________________

• Finding the Equation of a Line Given Two Points •

Direct
Teaching

How do we find the equation of a line if we are given two points on the line? Let's look at an example.

Example: Find the equation of the line going through the points (3, 2) and (4, 6).

Since we are given two points on the line, we can find the **slope** of the line. Recall that the **slope** is the *change* in y-coordinates divided by the *change* in x-coordinates.

$$\text{SLOPE} = \frac{\text{change in } y}{\text{change in } x} = \frac{6-2}{4-3} = \frac{4}{1} = 4$$

We can use *either* given point along with the slope to write the equation of the line in **point–slope form** since the line goes through *both* points. Let's use both points to write the equation of the line in **slope–intercept form** to show that either point will lead to the same equation.

Using (3, 2):	**Using (4, 6)**:
$y - 2 = 4(x - 3)$	$y - 6 = 4(x - 4)$
$y - 2 = 4x - 12$	$y - 6 = 4x - 16$
$y = 4x - 10$	$y = 4x - 10$

So, the equation of the line going through the points (3, 2) and (4, 6) written in **slope–intercept form** is $y = 4x - 10$.

Try this: **Find the slope, then write the equation of the line.**

1) Write the equation of the line going through the points (5, 8) and $(^-4, ^-10)$.

SLOPE = __________

SLOPE–INTERCEPT FORM: ____________________________

• Finding the Equation of a Line •

Find the slope, then write the equation of each line.

1) Write the equation of the line going through the points $(8, {}^{-}7)$ and $({}^{-}5, 8)$.

SLOPE = ____________

POINT–SLOPE FORM: ____

2) Write the equation of the line going through the points $(4, 0)$ and $({}^{-}4, 4)$.

SLOPE = ____________

SLOPE–INTERCEPT FORM: ____

3) Write the equation of the line going through the points $({}^{-}6, 7)$ and $(1, 9)$.

SLOPE = ____________

POINT–SLOPE FORM: ____

4) Write the equation of the line going through the points $({}^{-}8, 2)$ and $({}^{-}5, \frac{1}{2})$.

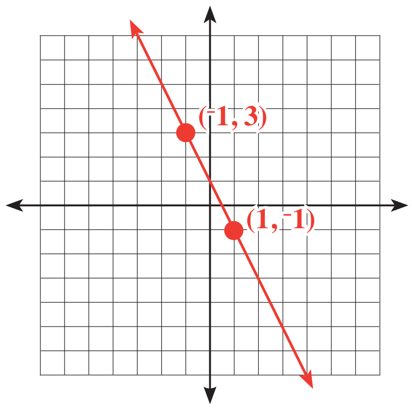
SLOPE = ____________

SLOPE–INTERCEPT FORM: ____

• Finding the Equation of a Line •

Write the equation of each line in POINT–SLOPE and SLOPE–INTERCEPT FORM.

1)



POINT–SLOPE FORM: ____________________

SLOPE–INTERCEPT FORM: ________________

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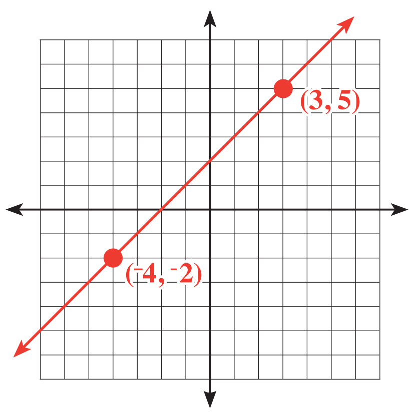
2)



POINT–SLOPE FORM: ____________________

SLOPE–INTERCEPT FORM: ________________

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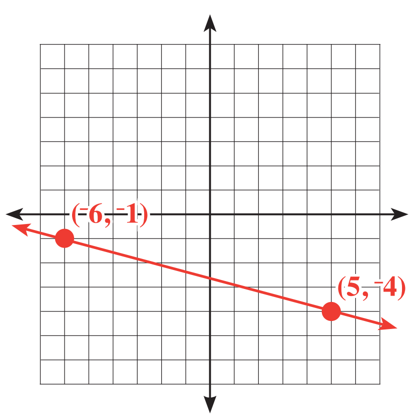
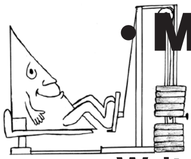
3)



POINT–SLOPE FORM: ____________________

SLOPE–INTERCEPT FORM: ________________



• Mastery Check: Finding the Equation of a Line •

☆

Write the equation of each line in any form.

1) Write the equation of the line with a slope of 3 that goes through the point $(2, 1)$.

____

2) Write the equation of the line with a slope of $\frac{1}{3}$ that goes through the point $({}^{-}4, 7)$.

____

3) Write the equation of the line going through the points $(1, 2)$ and $(3, {}^{-}5)$.

____

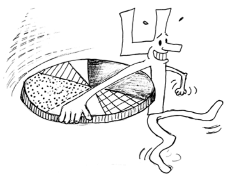
4) Write the equation of the line going through the points $(9, {}^{-}1)$ and $({}^{-}2, 6)$.

____

Challenge: **Write the equation of the line in SLOPE–INTERCEPT FORM.**

5) Find the equation of the line going through the points $(\boldsymbol{x}_1, \boldsymbol{y}_1)$ and $(0, b)$.

SLOPE–INTERCEPT FORM: ______________________________

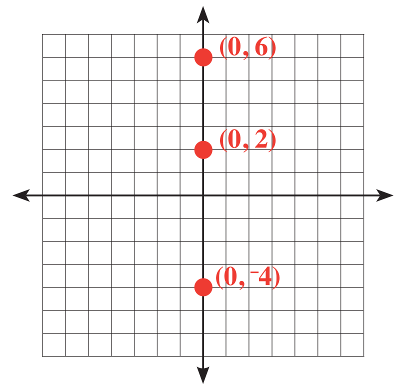


• The *x*– and *y*–Intercepts •

Direct Teaching

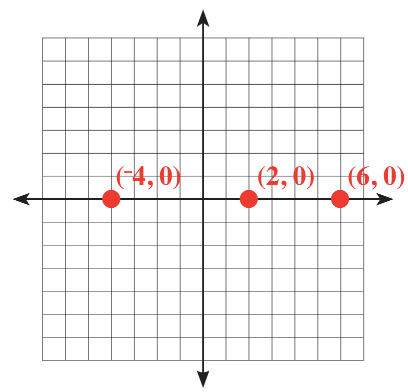
The ***y*–INTERCEPT** of a line is where the line intercepts or crosses the ***y***–axis.

Any point on the ***y***–axis has an ***x***–coordinate of 0. So, the ***y*–INTERCEPT** of a line will always have an ***x***–coordinate of 0.



Similarly, the ***x*–INTERCEPT** of a line is where the line intercepts or crosses the ***x***–axis.

Any point on the ***x***–axis has a ***y***–coordinate of 0. So, the ***x*–INTERCEPT** of a line will always have a ***y***–coordinate of 0.



Try these: **Circle the points that are x–intercepts. Put a box around the points that are y–intercepts.**

1) (5, 0) (0, 8) (2, 0) (0, $^{-}$3)

2) (7, 0) ($^{-}$12, 0) (4, 1) (0, 50)

3) Why do all ***x*–INTERCEPTS** have a ***y***–coordinate of 0? ____________________

____

4) Why do all ***y*–INTERCEPTS** have a ***x***–coordinate of 0? ____________________

____

• Finding the *x*– and *y*–Intercepts •

Direct Teaching

When we are given an equation written in **STANDARD FORM**, we can find the ***x***– and ***y*–INTERCEPTS** of the equation by substituting 0 into the equation for either variable.

EXAMPLE: What are the ***x***– and ***y*–INTERCEPTS** of the equation $4x - 3y = 12$?

We know the ***y*–INTERCEPT** of a line will always have an ***x***–coordinate of 0. So, we can substitute 0 for ***x*** and solve for ***y*** to find the ***y***-coordinate of the ***y*–INTERCEPT**.

Similarly, since the ***x*–INTERCEPT** of a line will always have a ***y***–coordinate of 0, we can substitute 0 for ***y*** and solve for ***x*** to find the ***x***-coordinate of the ***x*–INTERCEPT**.

$4(0) - 3y = 12$	$4x - 3(0) = 12$
$^{-}3y = 12$	$4x = 12$
$y = {}^{-}4$	$x = 3$
***y*–INTERCEPT**: $(0, {}^{-}4)$	***x*–INTERCEPT**: $(3, 0)$

Try these: **Identify the coordinates for the *x*–intercept and *y*–intercept of the equations.**

1) $x - y = 15$

***x*–INTERCEPT** = ____________ ***y*–INTERCEPT** = ____________

2) $6x - 4y = {}^{-}12$

***x*–INTERCEPT** = ____________ ***y*–INTERCEPT** = ____________

3) $3x + 2y = {}^{-}24$

***x*–INTERCEPT** = ____________ ***y*–INTERCEPT** = ____________

• Graphing Using Standard Form •

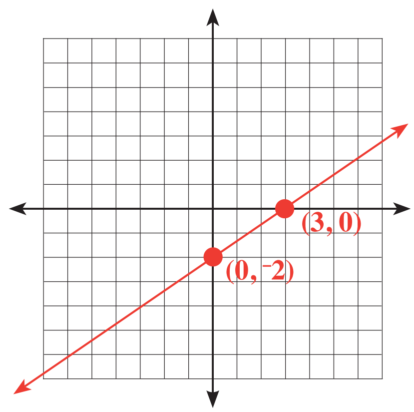
Direct Teaching

Since any two points define a line, we can graph an equation given in **STANDARD FORM** by plotting its x– and y–**INTERCEPTS**.

EXAMPLE: Graph the equation $2x - 3y = 6$.

We first find the x– and y–**INTERCEPTS** of the equation, which are $(3, 0)$ and $(0, {}^{-}2)$.

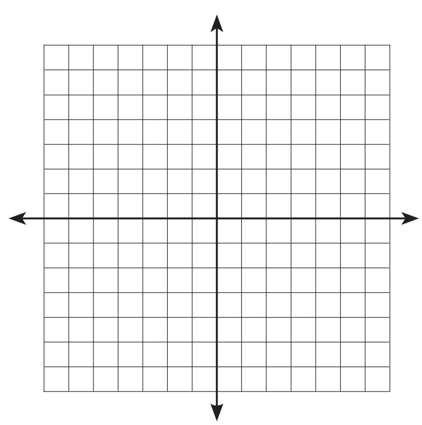
We can plot these two points and connect them to graph the equation $2x - 3y = 6$.

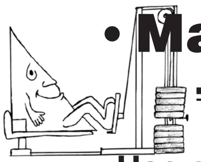


Try this: **Identify the coordinates for the *x*–intercept and *y*–intercept of the equation, then use a straight edge to graph.**

1) $6x - 4y = {}^{-}12$

x–INTERCEPT = ____________ y–INTERCEPT = ____________



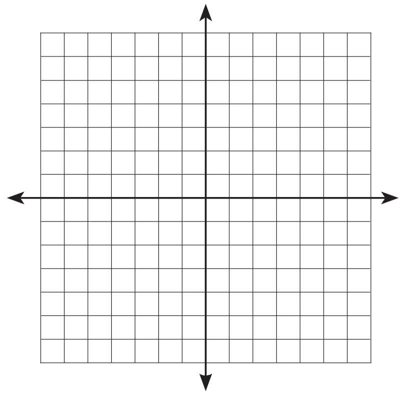


• Mastery Check: Graphing Using Standard Form •

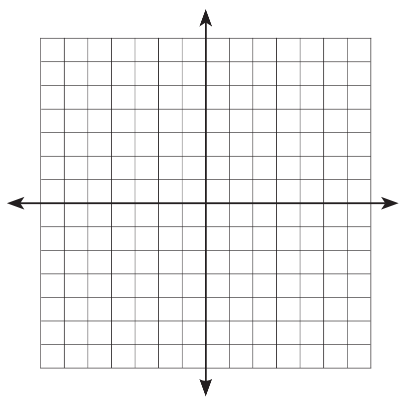
☆

Use a straight edge to graph the equations.

1) $6x + 2y = 12$

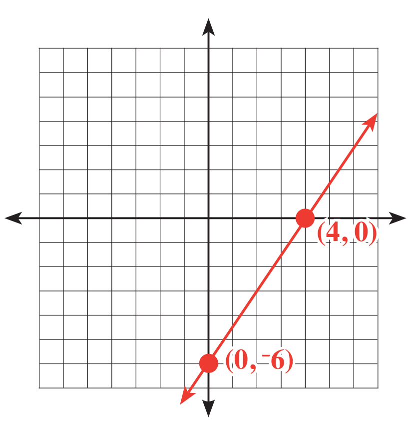


2) $3x - 9y = 9$



Challenge: Circle the equation(s) that correspond(s) to the graph.

3)



a) $6x + 4y = 24$

b) $3x - 2y = 12$

c) $6x - 4y = {}^{-}24$

d) $12x - 8y = 48$

• Slope and y–intercept •

An equation in **SLOPE–INTERCEPT FORM** is written as:

$$\boxed{y = mx + b}$$

where m is the slope of the line and b is the y-intercept of the line.

- The **SLOPE** of a line is the coefficient of x.
- The **y–INTERCEPT** of a line is given by the constant of the equation.

EXAMPLE: Find the slope and the coordinate of the y-intercept of the equation $y = \frac{2}{3}x - 7$.

The slope is the coefficient of x, so the slope is $\frac{2}{3}$.

The y–intercept is given by the constant of the equation, so the coordinate of the y–intercept is $(0, {}^{-}7)$.

Try these: **Find the slope and the coordinate of the *y*-intercept by inspection.**

1) $y = 2x + 1$

SLOPE = ____________

y–INTERCEPT = ____________

2) $y = {}^{-}10x$

SLOPE = ____________

y–INTERCEPT = ____________

3) $y = {}^{-}\frac{1}{2}x - 10$

SLOPE = ____________

y–INTERCEPT = ____________

4) $y = {}^{-}13$

SLOPE = ____________

y–INTERCEPT = ____________

5) $y = \frac{1}{2}x + 7$

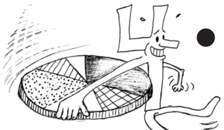
SLOPE = ____________

y–INTERCEPT = ____________

6) $y = 5x + 9$

SLOPE = ____________

y–INTERCEPT = ____________



• Graphing Using Slope-Intercept Form • ☆

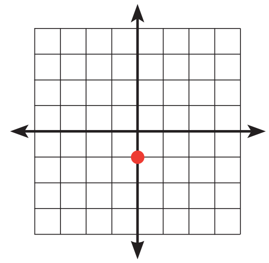
We can use the slope and ***y***–intercept to graph the line.

Example: Graph the equation $y = 4x - 1$ using the slope and ***y***-intercept.

Steps to Graph:

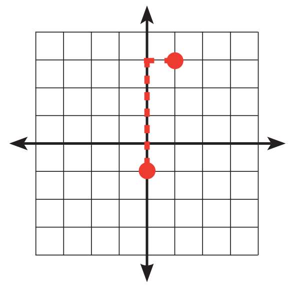
Step 1:

We begin by finding the ***y***–intercept and then plotting it. Since $b = {}^{-}1$, the ***y***–intercept is $(0, {}^{-}1)$.



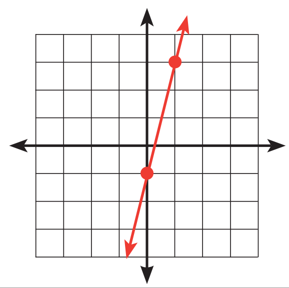
Step 2:

Next, we find the slope. Since $m = 4$, the slope is $4 = \frac{4}{1}$. Recall that the slope is the change in ***y*** divided by the change in ***x***. This means that we go up 4 (change in ***y***) and then go right 1 (change in ***x***).



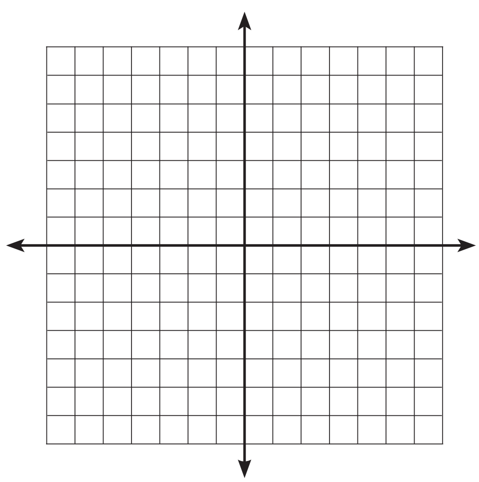
Step 3:

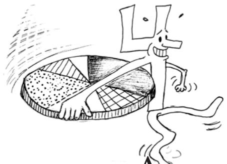
Now that we have two points, we can draw the line through the two points.



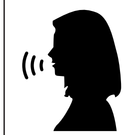
Try this: **Graph the equation by using the *y*-intercept and the slope.**

1) $y = \frac{3}{4}x + 1$





• Finding the x– and y–Intercepts •



Direct Teaching

An equation in **STANDARD FORM** is written as:

$$\boxed{Ax + By = C}$$

where A, B and C are integers and $A > 0$.

When we are given an equation written in **STANDARD FORM**, we can find the $\boldsymbol{x}$– and $\boldsymbol{y}$–intercepts of the equation by substituting 0 into the equation for either variable.

EXAMPLE: What are the $\boldsymbol{x}$– and $\boldsymbol{y}$–intercepts of the equation $4\boldsymbol{x} - 3\boldsymbol{y} = 12$?

We know the $\boldsymbol{y}$–intercept of a line will always have an $\boldsymbol{x}$–coordinate of 0. So, we can substitute 0 for $\boldsymbol{x}$ and solve for $\boldsymbol{y}$ to find the $\boldsymbol{y}$-coordinate of the $\boldsymbol{y}$–intercepts.

Similarly, since the $\boldsymbol{x}$–intercept of a line will always have a $\boldsymbol{y}$–coordinate of 0, we can substitute 0 for $\boldsymbol{y}$ and solve for $\boldsymbol{x}$ to find the $\boldsymbol{x}$-coordinate of the $\boldsymbol{x}$–intercepts.

$$4(0) - 3\boldsymbol{y} = 12$$
$$^{-}3\boldsymbol{y} = 12$$
$$\boldsymbol{y} = {}^{-}4$$

$\boldsymbol{y}$–**INTERCEPT**: $(0, {}^{-}4)$

$$4\boldsymbol{x} - 3(0) = 12$$
$$4\boldsymbol{x} = 12$$
$$\boldsymbol{x} = 3$$

$\boldsymbol{x}$–**INTERCEPT**: $(3, 0)$

***Try these*: Identify the coordinates for the $\boldsymbol{x}$–intercept and $\boldsymbol{y}$–intercept of the equations.**

1) $\boldsymbol{x} - \boldsymbol{y} = 5$

$\boldsymbol{x}$–**INTERCEPT** = ____________ $\boldsymbol{y}$–**INTERCEPT** = ____________

2) $3\boldsymbol{x} - 4\boldsymbol{y} = {}^{-}12$

$\boldsymbol{x}$–**INTERCEPT** = ____________ $\boldsymbol{y}$–**INTERCEPT** = ____________

• Graphing Using Standard Form •

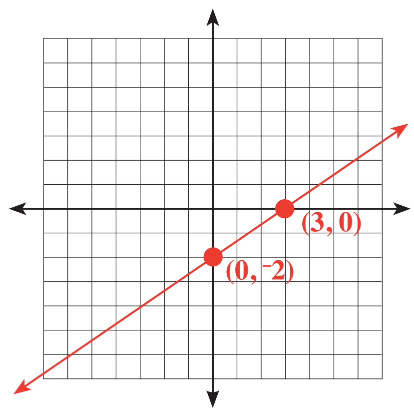
Direct Teaching

Since any two points define a line, we can graph an equation given in **STANDARD FORM** by plotting its x– and y–intercepts.

EXAMPLE: Graph the equation $2x - 3y = 6$.

We first find the x– and y–intercepts of the equation, which are $(3, 0)$ and $(0, ^-2)$.

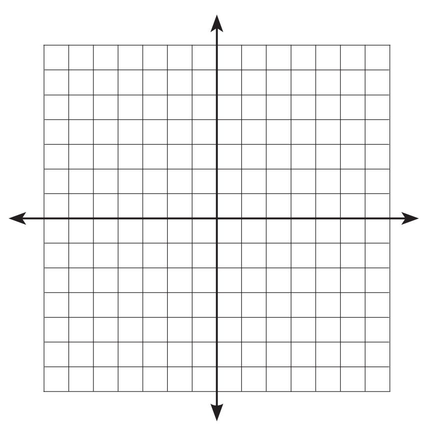
We can plot these two points and connect them to graph the equation $2x - 3y = 6$.



***Try this*: Identify the coordinates for the x–intercept and y–intercept of the equation, then use a straightedge to graph.**

1) $3x + 9y = {}^-18$

x–INTERCEPT = ____________ y–INTERCEPT = ____________



• Identifying the Slope and a Point on the Line •

An equation in **POINT–SLOPE FORM** is written as:

$$\boxed{y - y_1 = m(x - x_1)}$$

where m is the slope of the line that goes through the point (x_1, y_1).

EXAMPLE:
Identify the point on the line and the slope given by the equation $y + 3 = \frac{1}{3}(x + 2)$.

We can rewrite the equation as $y - (^-3) = \frac{1}{3}[x - (^-2)]$.

Now we can see that $x_1 = {^-2}$, $y_1 = {^-3}$, and $m = \frac{1}{3}$.

$$\begin{array}{ccccccccc} y & - & (^-3) & = & \frac{1}{3} & [\ x & - & (^-2) &] \\ & & \downarrow & & \downarrow & & & \downarrow & \\ y & - & y_1 & = & m & (\ x & - & x_1 &) \end{array}$$

So, the point $(^-2, {^-3})$ is a point on the line, and the slope of the line is $\frac{1}{3}$.

Identify the point on the line and the slope given by each equation.

1) $y - 3 = 3(x + 4)$

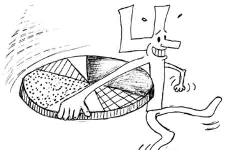
$(x_1, y_1) =$ $(^-4, 3)$ $\qquad m =$ 3

2) $y + 2 = {^-4}(x + 5)$

$(x_1, y_1) =$ ________ $\qquad m =$ ________

3) $y - 5 = (x + 10)$

$(x_1, y_1) =$ ________ $\qquad m =$ ________



• Graphing Using Point–Slope Form •

Direct Teaching

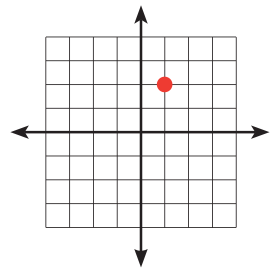
When given the equation of a line in POINT–SLOPE FORM [$y - y_1 = m(x - x_1)$], we can identify the slope and a point on the line. With this information, we can graph the line.

EXAMPLE: Graph the equation $y - 2 = {}^{-}2(x - 1)$.

Steps to Graph:

STEP 1:

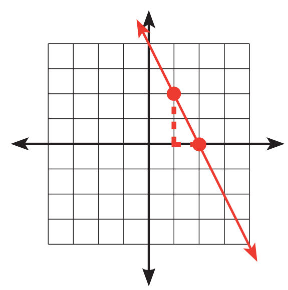
We begin by identifying a point on the line and plotting it. From the equation, we see that $x_1 = 1$ and $y_1 = 2$, so (1, 2) is a point on the line.



STEP 2:

Next, we identify the slope. Since $m = {}^{-}2$, the slope is $^{-}2$. Recall that the slope is the change in y divided by the change in x.
This means that we go down 2 (change in y) and then go right 1 (change in x).

Now that we have two points, we can draw the line through the two points.

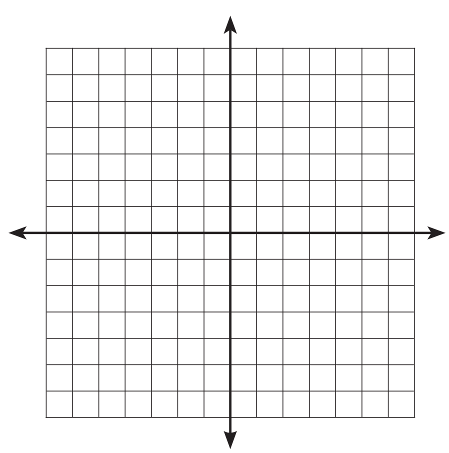


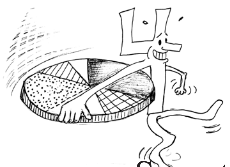
Try this: **Identify the point on the line and the slope given by the equation, then use a straightedge to graph the equation.**

1) $y - 1 = 3(x - 3)$

(x_1, y_1) = ___________

m = __________





• Best Method to Graph a Line •

Sometimes an equation of a line is not in one of the three forms we learned about. When this happens, we need to rewrite the equation in one of these forms before we can graph it.

Example: Determine which method is best to graph $7y = {}^{-}4x - 28$. Rewrite the equation in the correct form and graph.

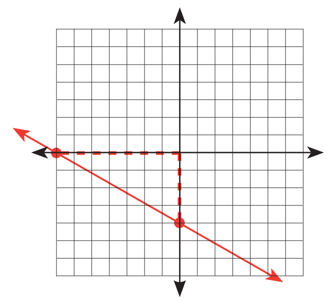
Slope–intercept form is a very common form to use to graph a line. $7y = {}^{-}4x - 28$ can be rewritten in slope–intercept form ($y = mx + b$) by dividing each term by 7. Since 28 divided by 7 is a whole number we will have a whole number y–intercept.

$$7y = {}^{-}4x - 28$$

$$y = {}^{-}\tfrac{4}{7}x - 4$$

$$m = {}^{-}\tfrac{4}{7}$$

y–intercept = $(0, {}^{-}4)$



Try these: **Determine the best method to graph the line. Rewrite the equation in the correct form and graph using a straightedge.**

1) $y = \frac{2}{3}(x + 1) - 2$

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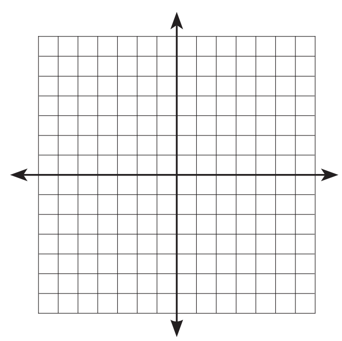

2) $6x = {}^{-}5y + 30$

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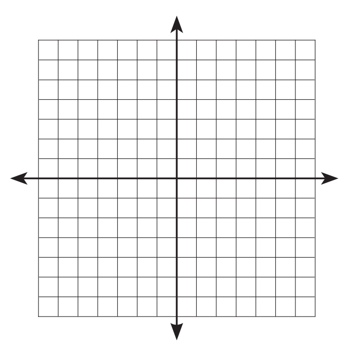
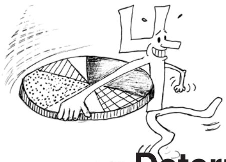


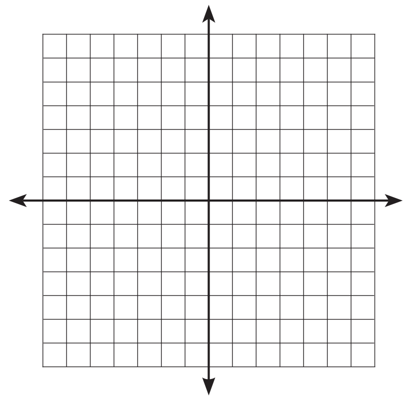


• Graphing Linear Equations •

☆

Determine the best method to graph the line. Rewrite the equation in the correct form and graph using a straightedge.

1) $y + 2 = \frac{2}{3}x - 2$



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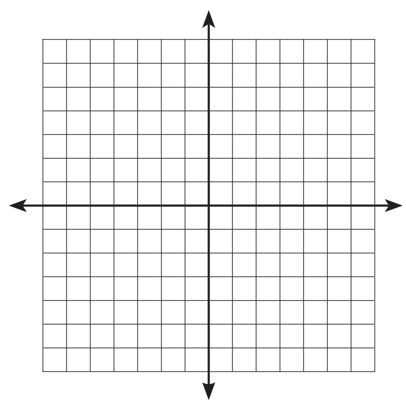
2) $4x + 7y = {}^{-}28$



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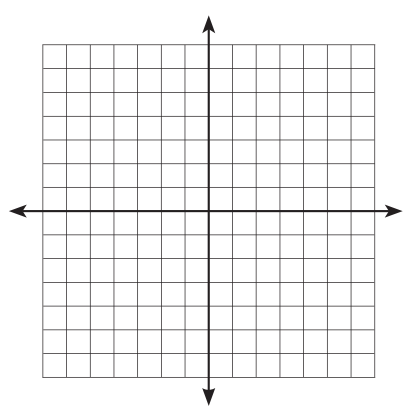
3) $y + 2 = \frac{3}{5}(x + 3) - 3$



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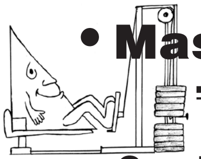
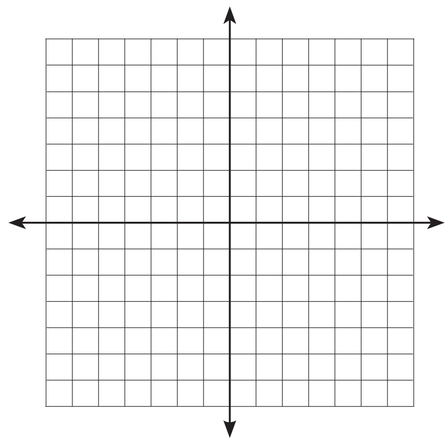


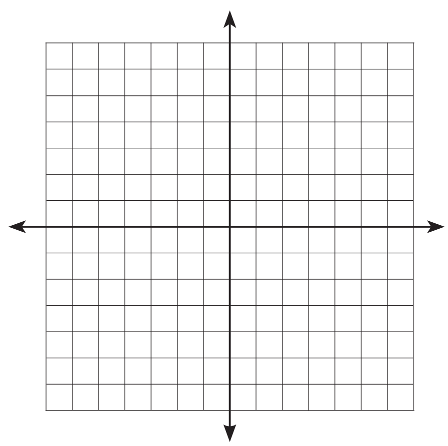
• Mastery Check: Graphing Linear Equations (Page 1 of 2) • ☆

Graph the equations.

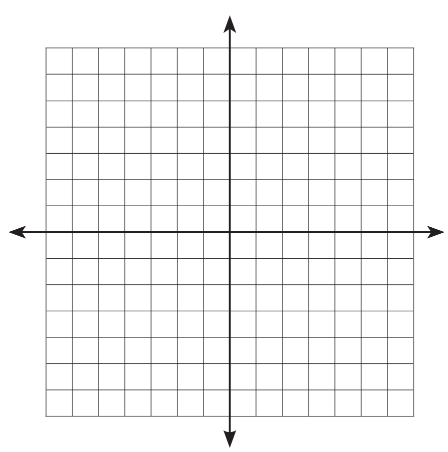
1) $2y = {}^{-}\frac{1}{2}x + 8$

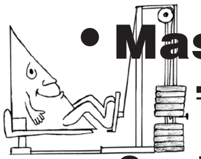


2) $6x + 2y = 12$



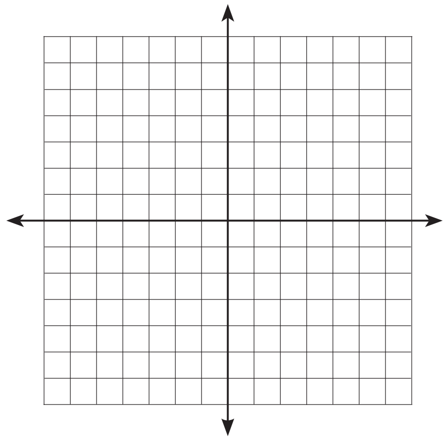
3) $y + 1 = \frac{3}{4}(x - 5) + 2$





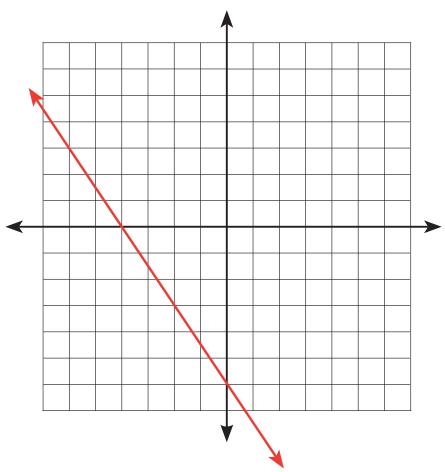
Graph the equations.

1) $y = {}^{-}\frac{1}{2}(x + 6)$



Challenge: **Write the equation of the line from the graph in slope–intercept, standard, and point–slope form.**

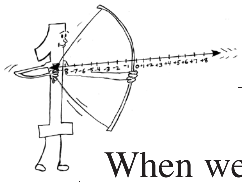
2)



SLOPE–INTERCEPT FORM: ______________________________

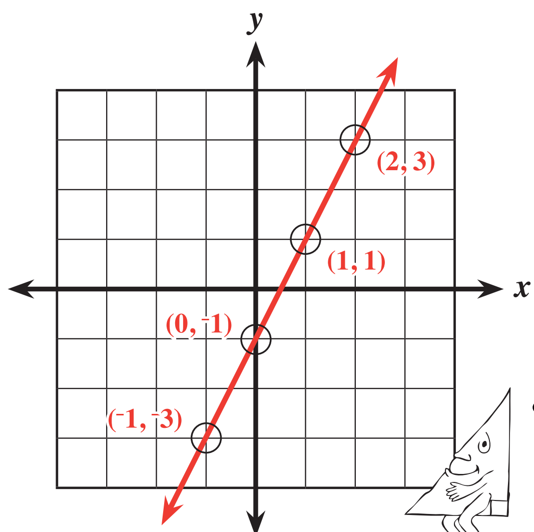
STANDARD FORM: ______________________________

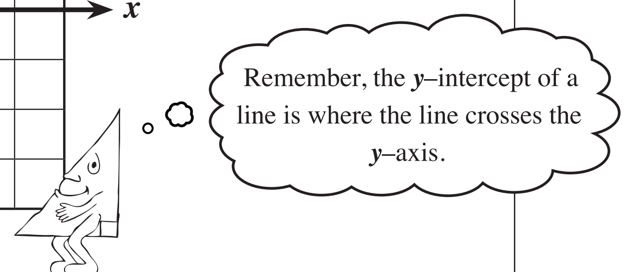
POINT–SLOPE FORM: ______________________________



• Identifying Points on a Graph •

When we are given the graph of a line, we can use the gridlines of the **COORDINATE PLANE** to help identify points on the line. We can easily identify coordinates wherever the vertical and horizontal gridlines intersect: . So, to identify points on a line, we find wherever the line and both gridlines all intersect: . A useful point to look for first is the *y*–intercept because it will help us find the equation of the line in **SLOPE–INTERCEPT FORM**. We've circled and labeled the points where the line and both gridlines intersect in the example below.

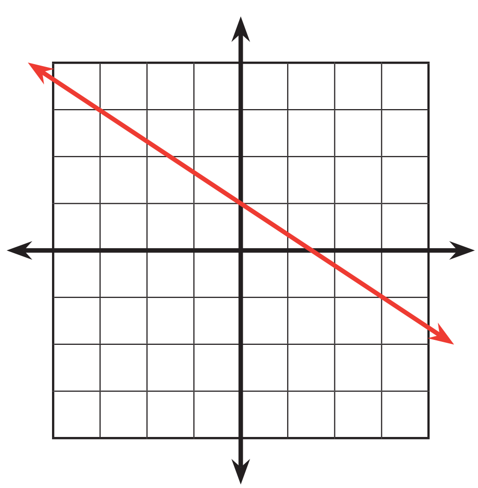




So, the points $(2, 3)$, $(1, 1)$, $(0, {}^{-}1)$, and $({}^{-}1, {}^{-}3)$ are all points on the line. Additionally, $(0, {}^{-}1)$ is the *y*–intercept of the line.

***Try this*: Plot and label all the points on the line that intersect the gridlines. Identify the *y*–intercept.**

1)

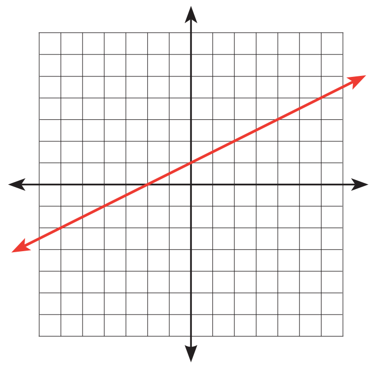


y–INTERCEPT: ______________

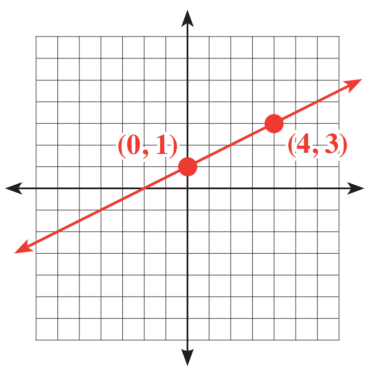
• Finding the Equation of a Line •

When we are given the graph of a line, we can identify the ***y***–intercept and find the slope of the line to find the equation of the line.

Example: What is the equation of the given line in **slope–intercept form**?



First we identify the ***y***–intercept, which is (0, 1). Then, we identify a second point, (4, 3), in order to find the slope. With the slope and the ***y***–intercept, we can find the equation of the line in **slope–intercept form**.



$$\text{slope} = \frac{3-1}{4-0} = \frac{2}{4} = \frac{1}{2}$$

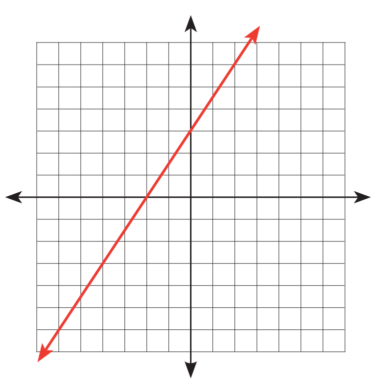
y–intercept = (0, 1)

$$y = mx + b$$

$$y = \frac{1}{2}x + 1$$

Try this: **Plot and label the *y*–intercept and a second point on the line.**

1)



***y*–intercept**: ______________

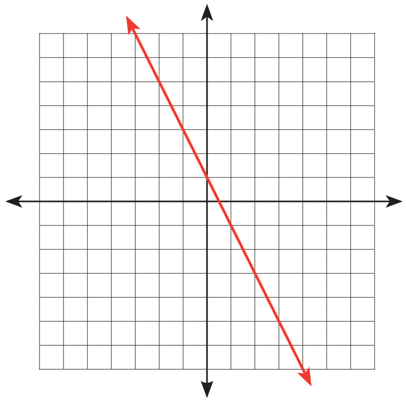
What is the slope of the line? __________

Write the equation of the line in **slope–intercept form**: __________________

• Finding the Equation of a Line •

For each graph, plot and label the *y*–intercept and a second point on the line.

1)



y–INTERCEPT: ______________

What is the slope of the line? __________

Write the equation of the line in SLOPE–INTERCEPT FORM: __________________

Use a third point to check your line for reasonableness.

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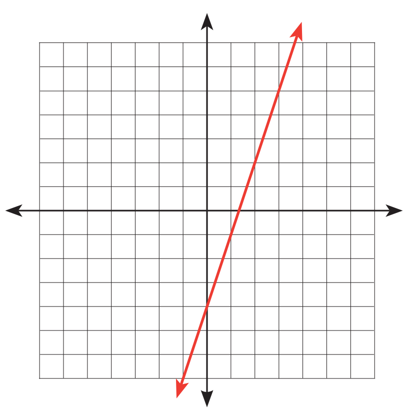
2)



y–INTERCEPT: ______________

What is the slope of the line? __________

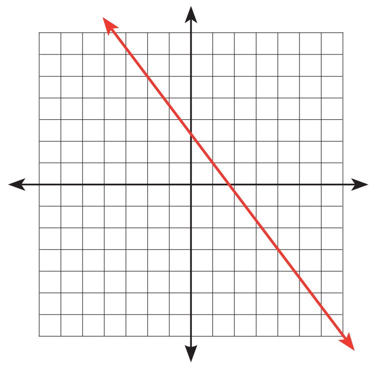
Write the equation of the line in SLOPE–INTERCEPT FORM: __________________

Use a third point to check your line for reasonableness.

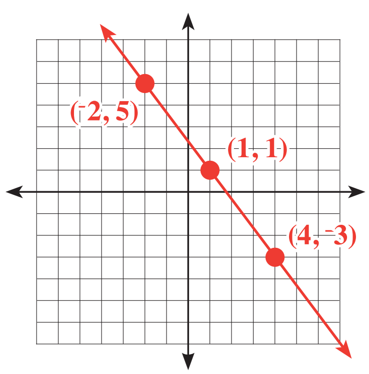
• Finding the Equation of a Line •

When we are given the graph of a line and we cannot easily identify the y–intercept, we identify any two points and use **POINT–SLOPE FORM** to find the equation of the line.

EXAMPLE: What is the equation of the given line in **POINT–SLOPE FORM**?



We identify two points, (⁻2, 5) and (1, 1). Then we can calculate the slope. With the slope and a point, we can write the equation of the line in **POINT–SLOPE FORM**. We'll check for reasonableness with a third point, (4, ⁻3)



slope $= \dfrac{1-5}{1-{}^{-}2} = -\dfrac{4}{3}$ $\qquad y - y_1 = m(x - x_1)$

Using the point (1, 1): $\quad y - 1 = -\frac{4}{3}(x-1)$

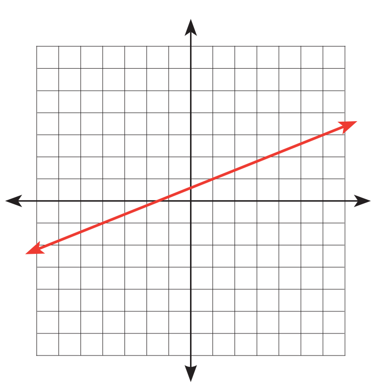
Check using (4, ⁻3): $\quad {}^{-}3 - 1 = -\frac{4}{3}(4-1)$

$${}^{-}4 = {}^{-}4 \ \checkmark$$

So, the equation of the line in **POINT–SLOPE FORM** is $y - 1 = -\frac{4}{3}(x-1)$.

Try this: **Plot and label two points on the line. Write the equation of the line in POINT–SLOPE FORM. Use a third point to check for reasonableness.**

1)



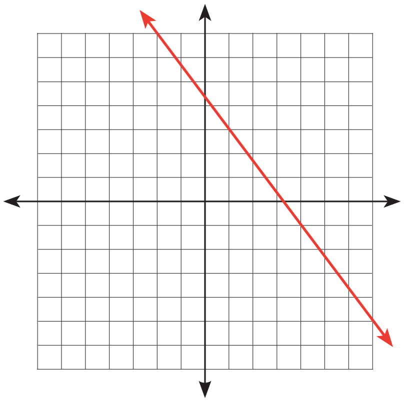
What is the slope of the line? __________

POINT–SLOPE FORM: ____________________

• Finding the Equation of a Line •

For each graph, plot and label all the points on the line that intersect the gridlines.

1)



What is the slope of the line? __________

Write the equation of the line in **POINT–SLOPE FORM**: ____________________

Use a point to check your line for reasonableness.

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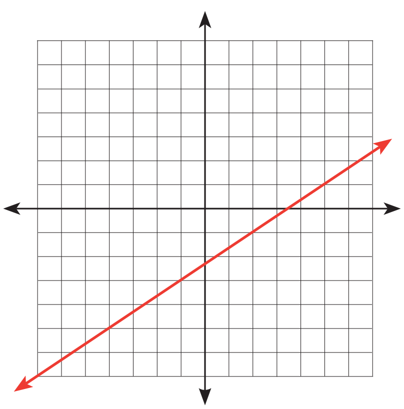
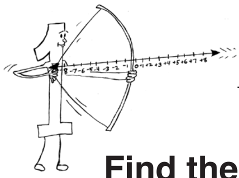
2)



What is the slope of the line? __________

Write the equation of the line in **POINT–SLOPE FORM**: ____________________

Use a point to check your line for reasonableness.

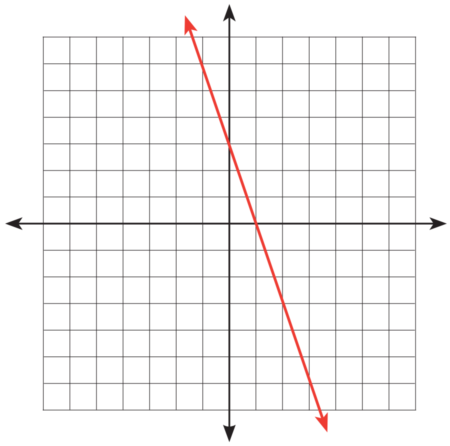


• Finding the Equation of a Line •

☆

Find the equation of the line in any form. Check for reasonableness.

1)



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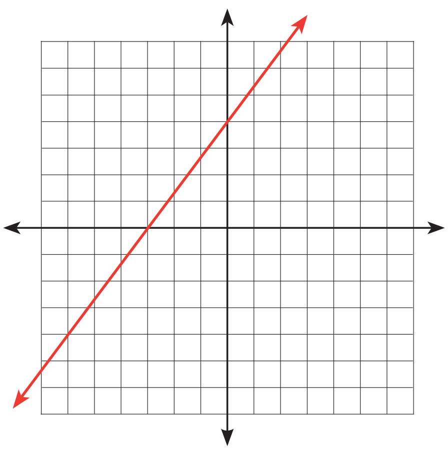
2)



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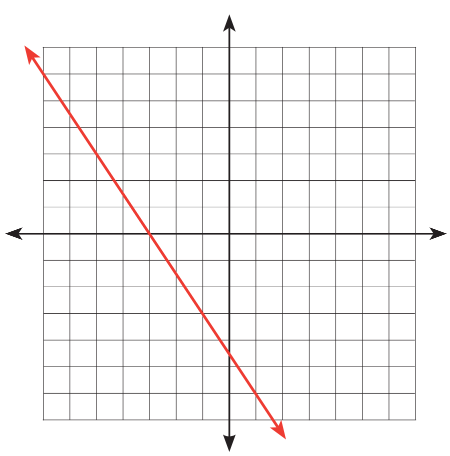
3)



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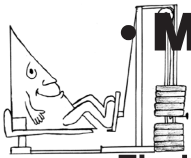
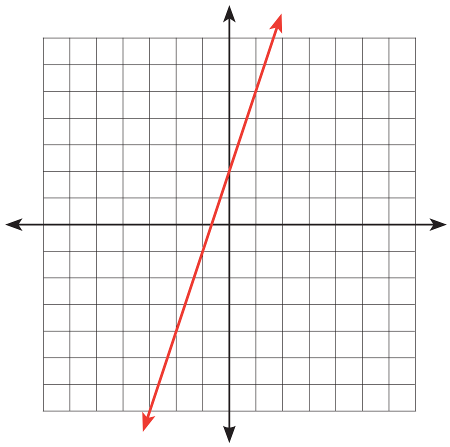


• Mastery Check: Finding the Equation of a Line •

Find the equation of the line in any form. Check for reasonableness.

1)



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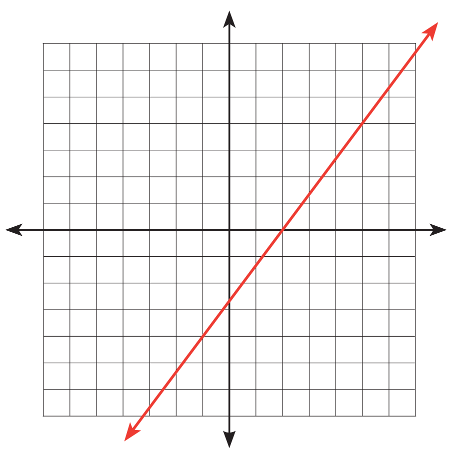
2)



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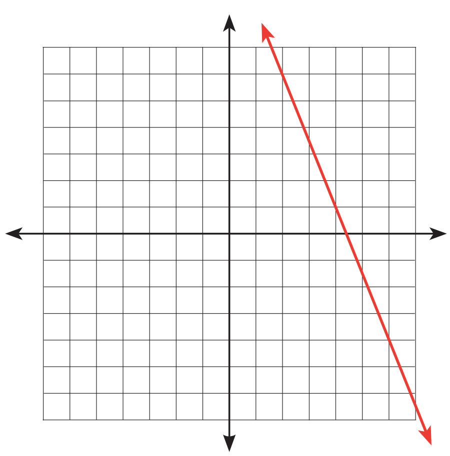
3)



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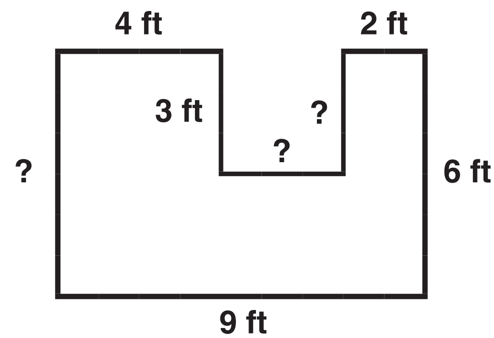
• Missing Dimensions of Composite Figures •

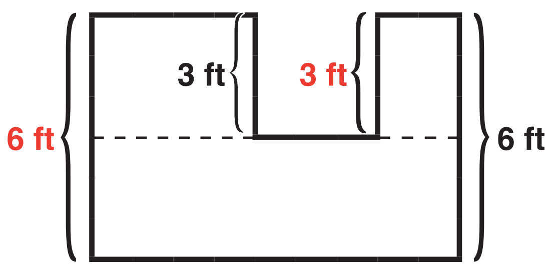
Direct Teaching

Sometimes we are given a composite figure with some missing dimensions. We can use what we are given to figure out the missing dimensions.

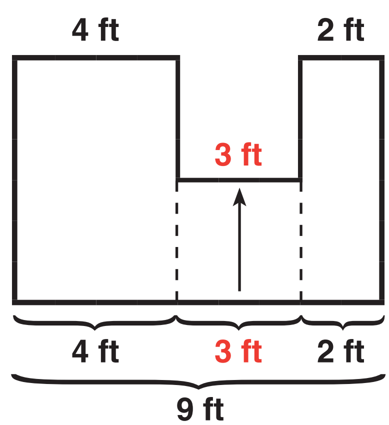
Example: Find the missing dimensions of the following figure.



We can imagine the figure as three separate rectangles to help us find its dimensions.



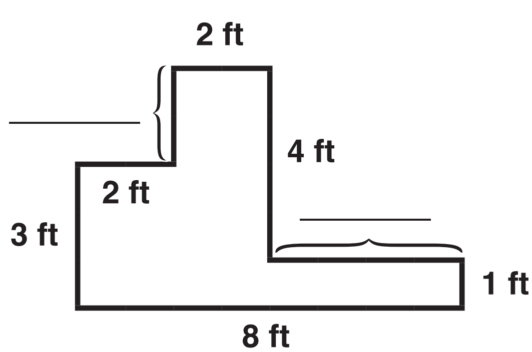
We can find some of the missing dimensions by inspection.

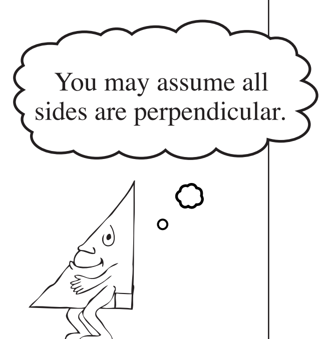


Since we know the bottom of the figure measures 9 feet, we can figure out that the length of the remaining missing dimension is 3 feet.

Try this: **Find the missing dimensions of the following figure.**

1)

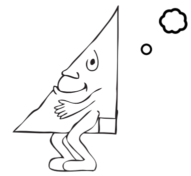




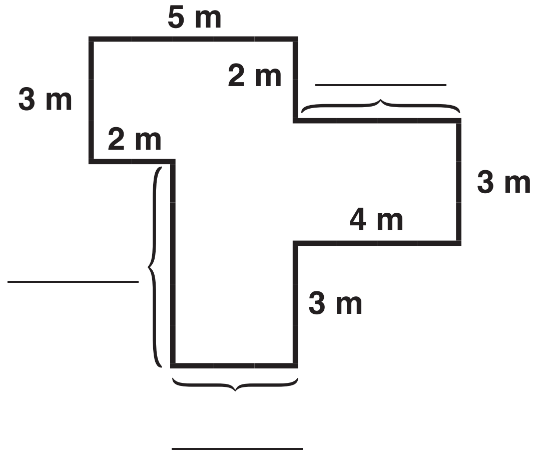
• Missing Dimensions of Composite Figures • ☆

Find the perimeters of each figure.

You may assume all sides are perpendicular.

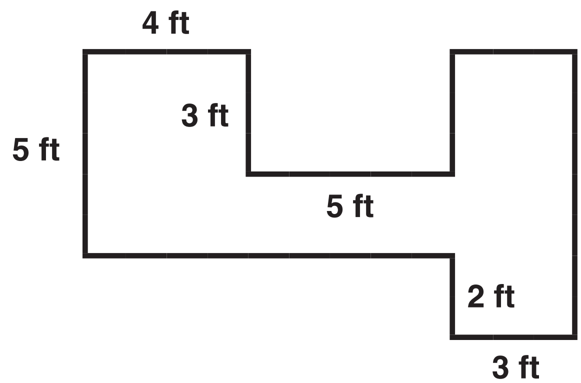


1)



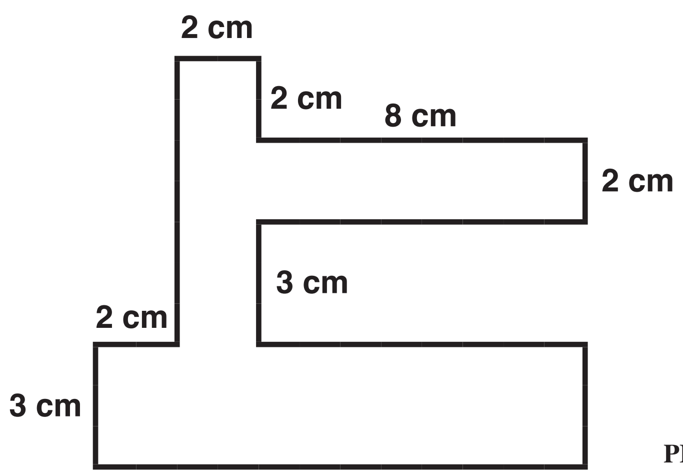
PERIMETER = ______________

2)



PERIMETER = ______________

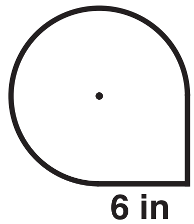
3)



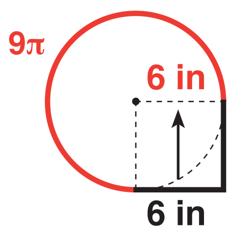
PERIMETER = ______________

• Perimeter of Composite Figures •

Example: Find the perimeter of the following figure.

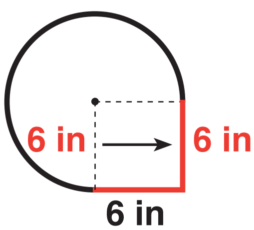


We can imagine the figure as a three–quarter circle and a square. To find the perimeter of the three–quarter circle, we will first find the circumference of the whole circle with the same radius.



We can figure out the radius of the circle from the bottom edge of the square (6 in). So, the circumference of the whole circle is $2\pi(6) = 12\pi$ in.

The length of the curved part of the figure is three–quarters of the circumference, which is $\frac{3}{4}(12\pi \text{ in}) = 9\pi$ in.

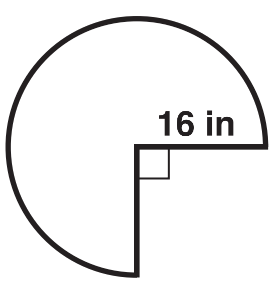


We can then add the lengths of the two sides of the square. So the perimeter of the whole figure is

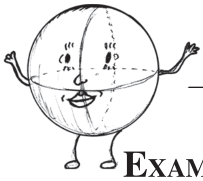
$$9\pi + 2(6) = (12 + 9\pi) \text{ in}.$$

Try this: **Find the perimeter of the following figure. Leave answer in exact form.**

1)

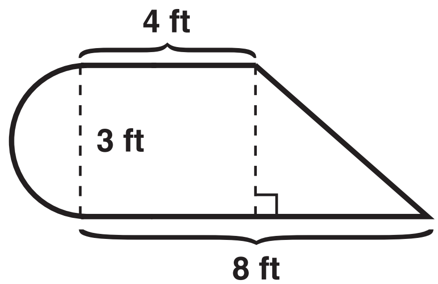


PERIMETER = ____________________

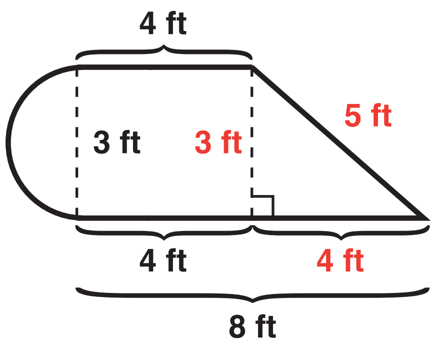


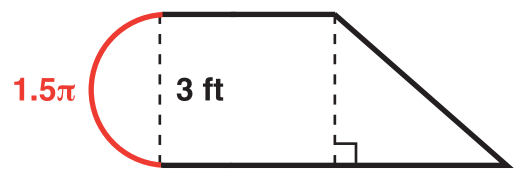
• Perimeter of Composite Figures •

Example: Find the perimeter of the following figure.



First find the missing dimensions.





Since the triangle part of the figure is a right triangle, we can use the **Pythagorean Theorem** to find the length of the hypotenuse.

$$3^2 + 4^2 = c^2,\ c = 5 \text{ ft}$$

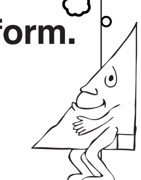
We can find the perimeter of the semicircle part of the figure by finding half the circumference of a circle with diameter of 3 feet.

$$\frac{1}{2} \bullet \pi \bullet d = \frac{1}{2} \bullet \pi \bullet 3 = 1.5\pi \text{ ft}$$

Adding up all the sides of the figure, we find that the perimeter is:

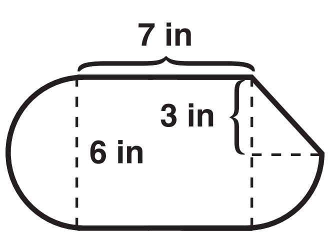
$$4 + 5 + 8 + 1.5\pi = (17 + 1.5\pi) \text{ ft}$$

You may assume all angles that look right are right angles.

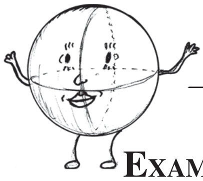


Try this: **Find the perimeter of the following figure. Leave answer in exact form.**

1)

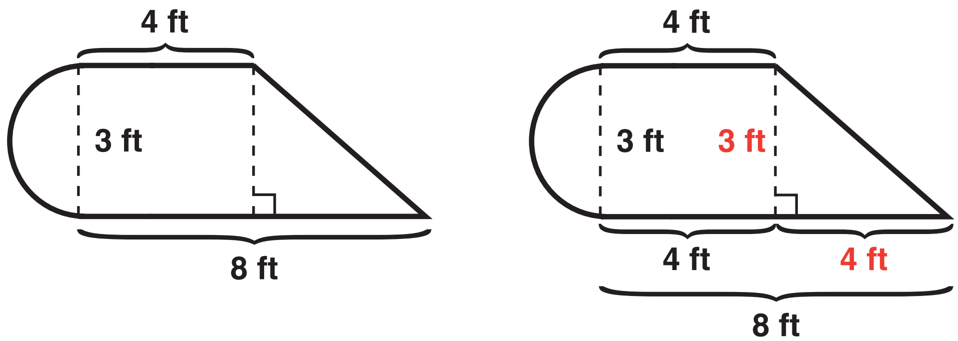


PERIMETER = ______________________

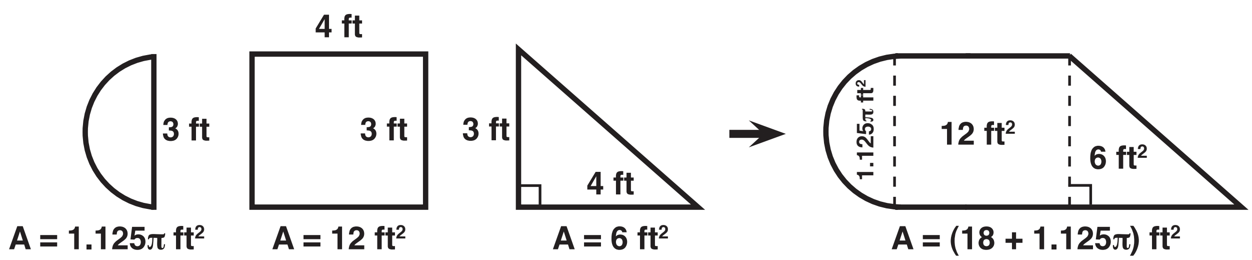


• Area of Composite Figures •

Example: Find the area of the following figure.



To find the area of the whole composite figure, we add together the areas of each part of the figure. We can imagine the figure broken up as a semicircle, a rectangle, and a right triangle.



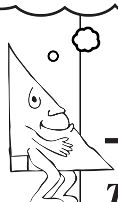
The area of the semicircle: $\frac{1}{2} \bullet \pi \bullet r^2 = \frac{1}{2} \bullet \pi \bullet (1.5)^2 = 1.125\pi \text{ ft}^2$

The area of the rectangle: $l \bullet w = 4 \bullet 3 = 12 \text{ ft}^2$

The area of the triangle: $\frac{1}{2} \bullet b \bullet h = \frac{1}{2} \bullet 4 \bullet 3 = 6 \text{ ft}^2$

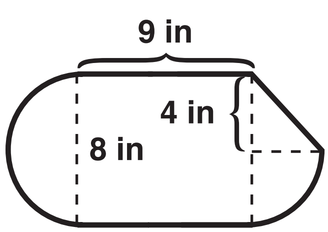
So, the area of the whole figure is $12 + 6 + 1.125\pi = (18 + 1.125\pi) \text{ ft}^2$.

You may assume all angles that look right are right angles.

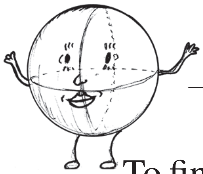


Try this: **Find the area of the following figure. Leave answer in exact form.**

1)



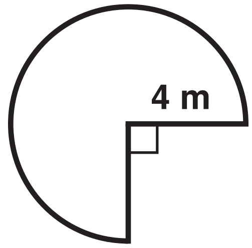
AREA = ______________________



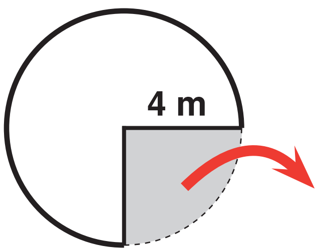
• Area of a Fraction of Circle •

To find the area of a composite figure, we can also subtract a part from a larger whole.

EXAMPLE: Find the area of the following figure.



Here we have a three–quarter circle with a radius of 4 m. We can imagine this composite figure as a circle with a quarter circle missing from it.



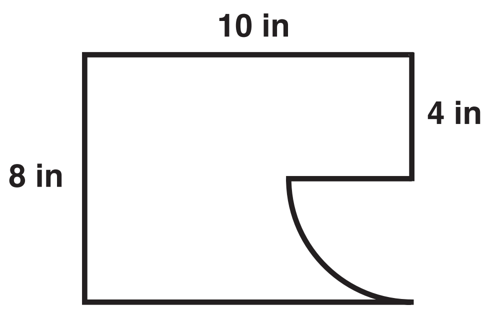
The area of the whole circle is $\pi(4)^2 = 16\pi \text{ m}^2$.

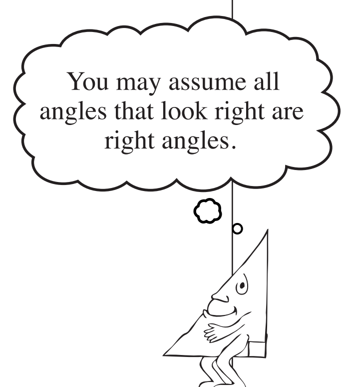
So, the area of the composite figure is the area of the whole circle ($16\pi \text{ m}^2$) minus the area of the quarter circle ($\frac{1}{4} \bullet 16\pi$).

$$\Rightarrow 16\pi - (\tfrac{1}{4})16\pi = (\tfrac{3}{4})16\pi = 12\pi \text{ m}^2.$$

Try this: **Find the area of the following figure. Leave answer in exact form.**

1)



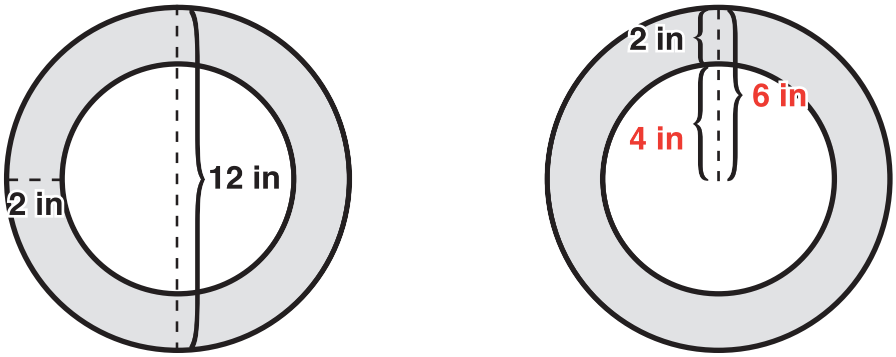


________ – ________ = ________

AREA OF RECTANGLE	AREA OF QUARTER CIRCLE	AREA OF COMPOSITE FIGURE

• Area of Composite Figures •

EXAMPLE: Find the area of the shaded region.



Since the diameter of the larger circle is 12 in, we know that its radius is 6 in. We can find the radius of the smaller circle (4 in) by subtracting the width of the shaded region (2 in) from the radius of the larger circle (6 in).

Here, the shaded region is a part of the whole larger circle. So, to find the area of the shaded region, we can subtract the area of the smaller circle from the area of the larger circle.

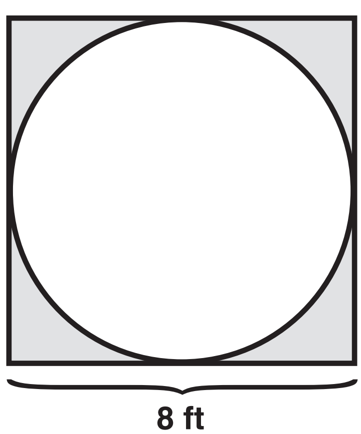
The area of the larger circle: $\pi \bullet \boldsymbol{r}^2 = \pi \bullet 6^2 = 36\pi \text{ in}^2$

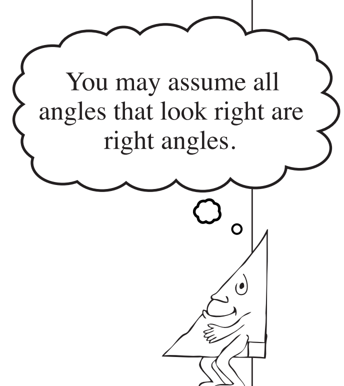
The area of the smaller circle: $\pi \bullet \boldsymbol{r}^2 = \pi \bullet 4^2 = 16\pi \text{ in}^2$

So, the area of the shaded region is $36\pi - 16\pi = 20\pi \text{ in}^2$.

Try this: **Find the area of the shaded region. Leave answer in exact form.**

1)

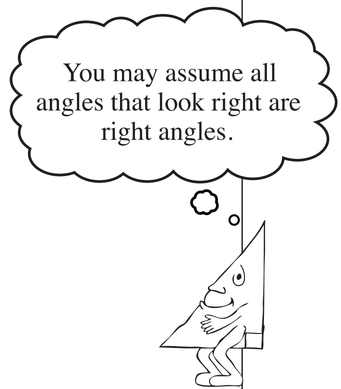




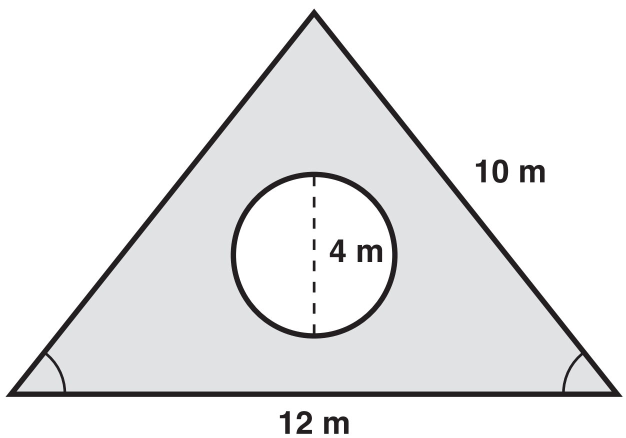
AREA = ____________________

• Area of Composite Figures •

Find the areas of the shaded regions. Leave answers in exact form.

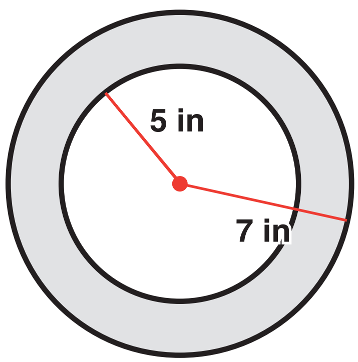


1)



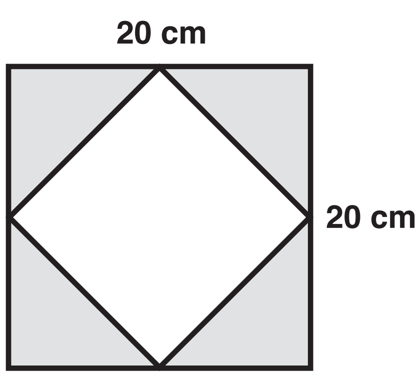
AREA = ____________________

2)

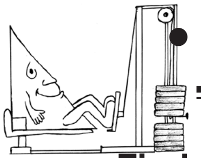


AREA = ____________________

3)



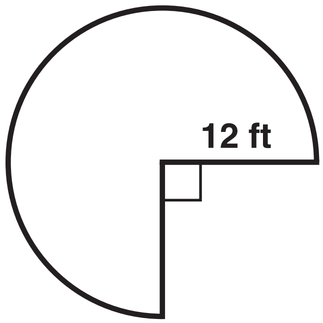
AREA = ____________________



• Mastery Check: Composite Figures •

Find the perimeter and area of the figure. Leave answers in exact form.

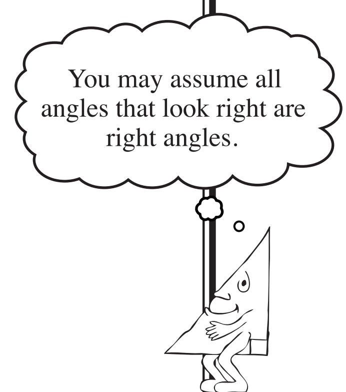
1)



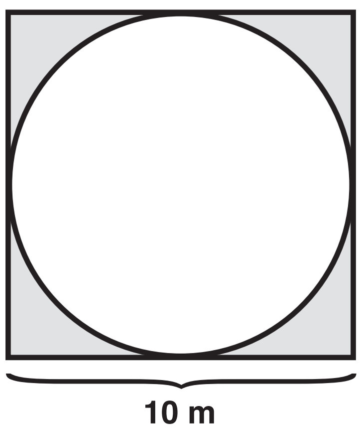
PERIMETER = ______________________

AREA = ______________________

Find the area of the shaded region. Leave answer in exact form.



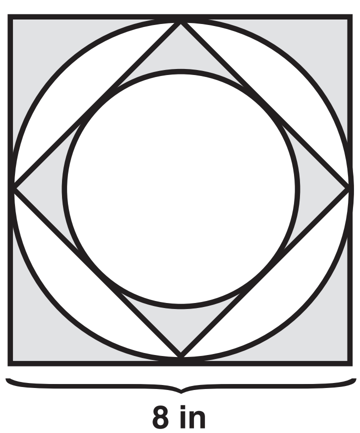
2)



AREA = ______________________

Challenge: Find the area of the shaded region. Leave answer in exact form.

3)



AREA = ______________________